Unit 3

Zeroth and first law of thermodynamics formulated in terms suitable for atmospheric sciences

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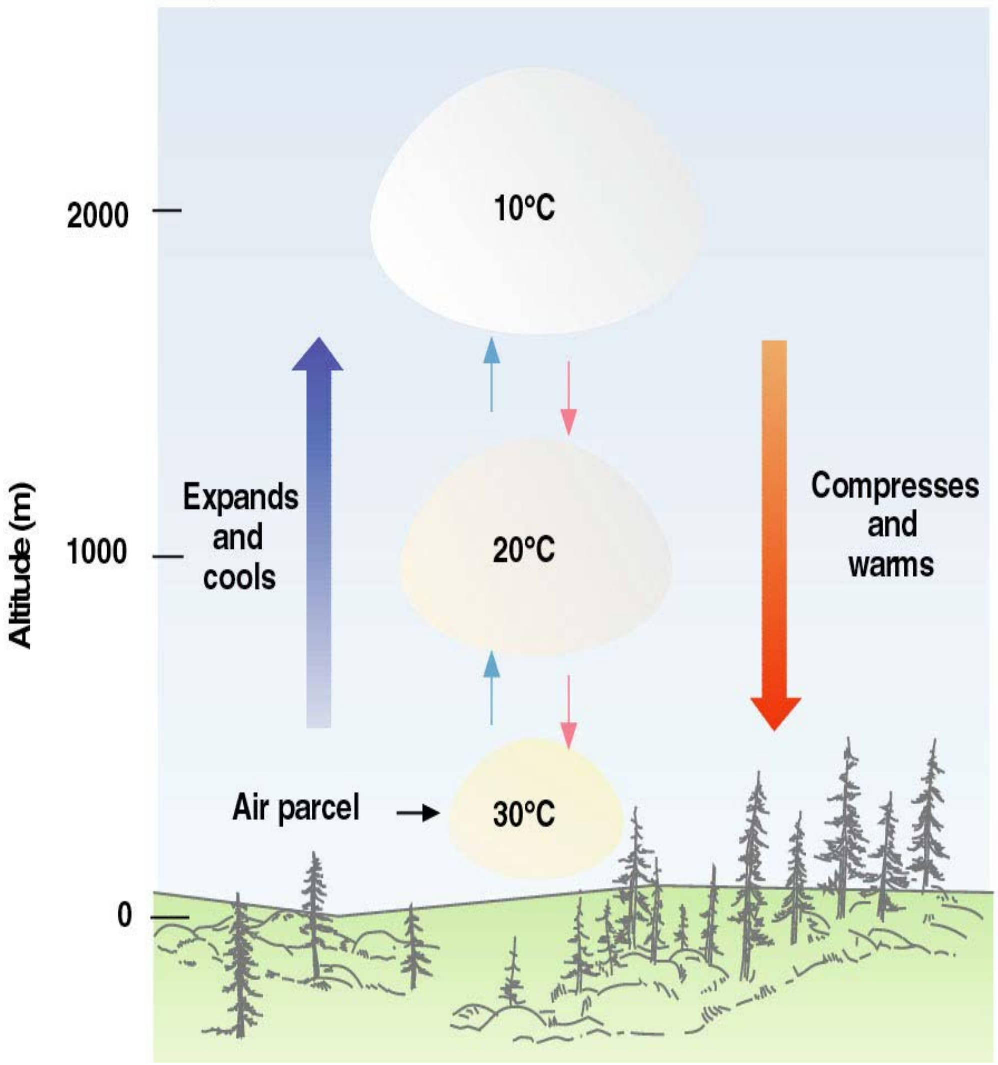
First law of thermodynamics

$$du = \delta q - \delta w + \sum_i \mu_i dn_i$$

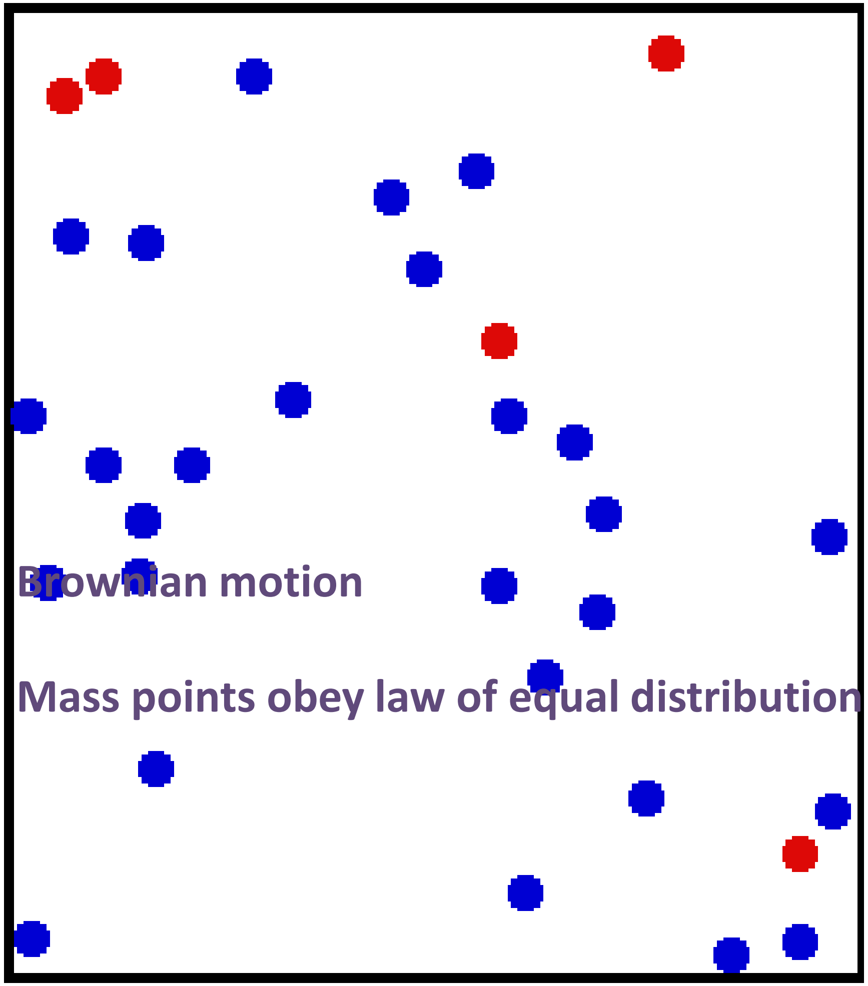
$$dE_{total} = dU + d(\frac{v^2}{2}) + d\Phi + \delta Q$$

Expansion & contraction



http://www.phys.ufl.edu/courses/met1010/chapter7-1.pdf

Kinetic theory of heat: U=const.·T



Brownian motion

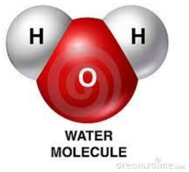
Mass points obey law of equal distribution

$$\overline{U} = \frac{m\overline{v^2}}{2} = \frac{3}{2}kT$$

From: http://en.wikipedia.org/wiki/File:Translational_motion.gif

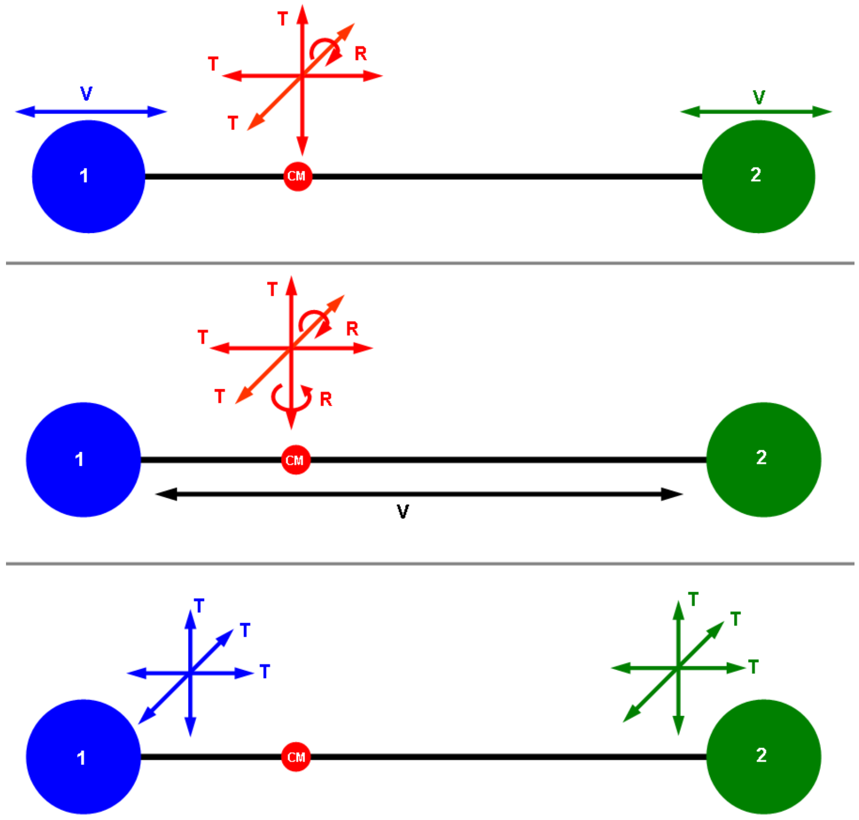
of degrees of freedom for molecules differs

CO_2



$$\overline{U} = \frac{1}{2} f n R^* T = \frac{f}{2} k T$$

Recall: Temperature of thermosphere is determined from kinetic theory of heat!



http://upload.wikimedia.org/wikipedia/commons/4/43/Degrees_of_freedom_%28diatomic_molecule%29.png

Specific heat

Definitions: $$c_v := \frac{\delta q}{dT}\Big|_v = \frac{T\,ds}{dT}\Big|_v$$

$$c_p := \frac{\delta q}{dT}\Big|_p = \frac{T\,ds}{dT}\Big|_p$$

1^{st} law of thermodynamics : $\delta q = Tds = du + pdv$

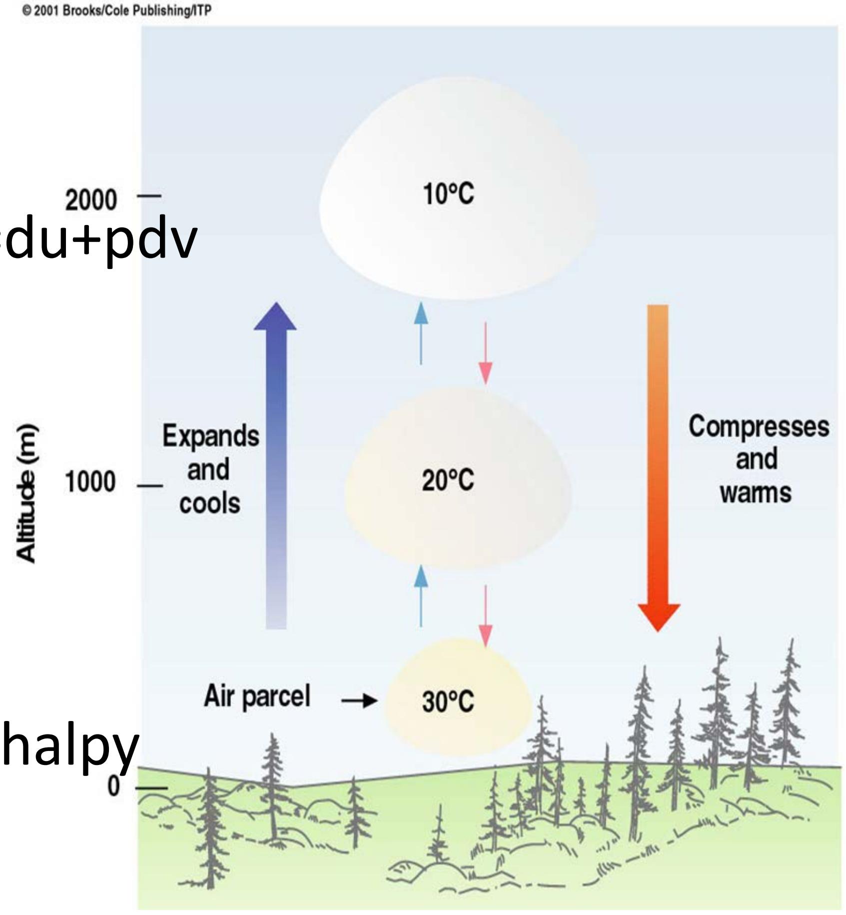
If v=0

$\Rightarrow du = c_v dT$

$\Rightarrow$ If du=0 $\Rightarrow$ u(T)

If p=0 with Tds=dh-vdp

$\Rightarrow dh = c_p dT$ with h=u+pv specific enthalpy



After integration: $u = c_v T$, $h = c_p T$

R, c_p and c_v are related

$Tds=c_v dT+pdv=c_v dT+d(pv)-vdp$

With Eq. of state

$$c_v dT+d(pv)-vdp=c_v dT+RdT-vdp=(c_v+R)dT-vdp$$

Comparison with $Tds=c_p dT-vdp$ yields

$c_p-c_v=R$

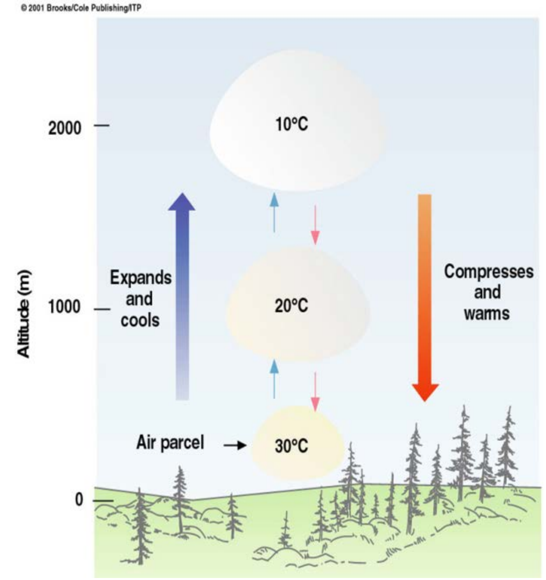
Air parcel:

c_p=1004 J/(kgK), c_v= 717J/(kgK), R_d=287 J(kgK)

$(c_p-c_v)\ c_p=R_d/c_p=\kappa=0.286$ Poisson-constant

$c_p/c_v=1.4$

Potential temperature



- Adiabatic ascend: $\delta Q = Tds = 0$

$\Rightarrow$ expanses $dv>0$

$\Rightarrow$ performs work pdv

$\Rightarrow$ $du<0$, $dT<0$

Question: $dT/dz=?$

$$c_p dT = \frac{1}{\rho} dp$$

$$c_p dT = \frac{R_d T}{p} dp$$

$$c_p \frac{dT}{T} - \frac{R_d dp}{p} = 0 \quad \text{or} \quad c_p d(lnT) - R_d d(lnp) = 0$$

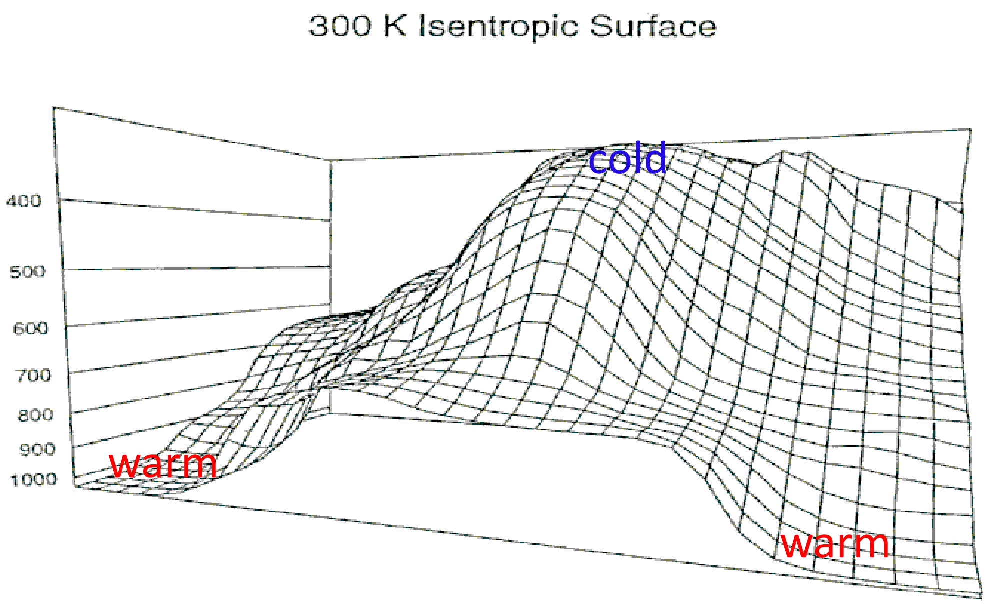
$$c_p lnT - R_d lnp = c_p lnT_o - R_d lnp_o = constant$$

$$ln\frac{T_o}{T} = \frac{R_d}{c_p} ln\frac{p_o}{p}$$

$$\Theta = T\left(\frac{p_o}{p}\right)^{\frac{R_d}{c_p}}$$

Potential temperature

3D isentropic topography can be used for assessment of trace gas/particle origin area



www.crh.noaa.gov/lsx/science/pdfppt/Isenanalysis.ppt

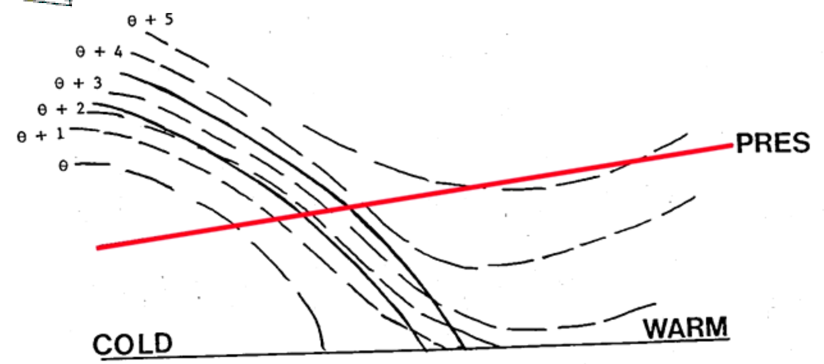
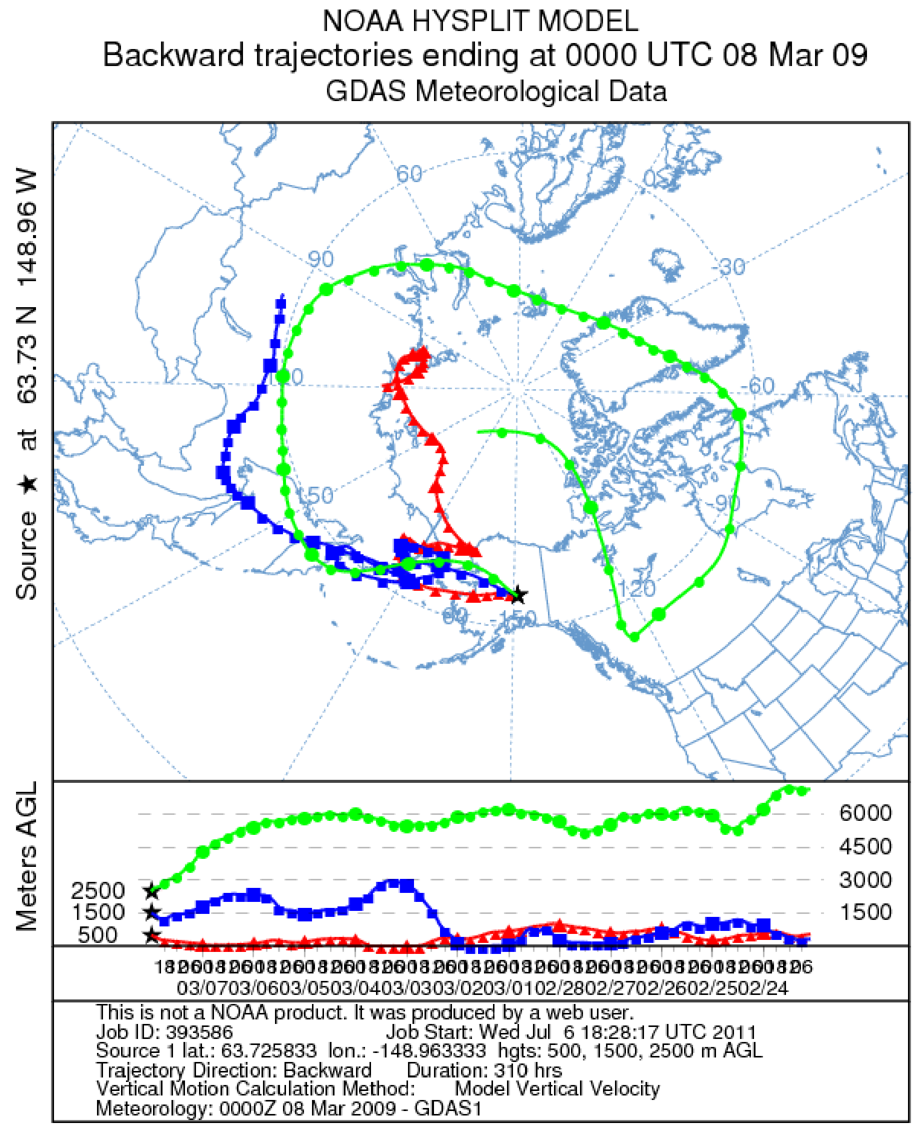


Figure 4. Schematic depiction of isentropic surfaces (dashed lines) and representative pressure surface (solid line) in the vicinity of a cold frontal zone (bold, curved lines).

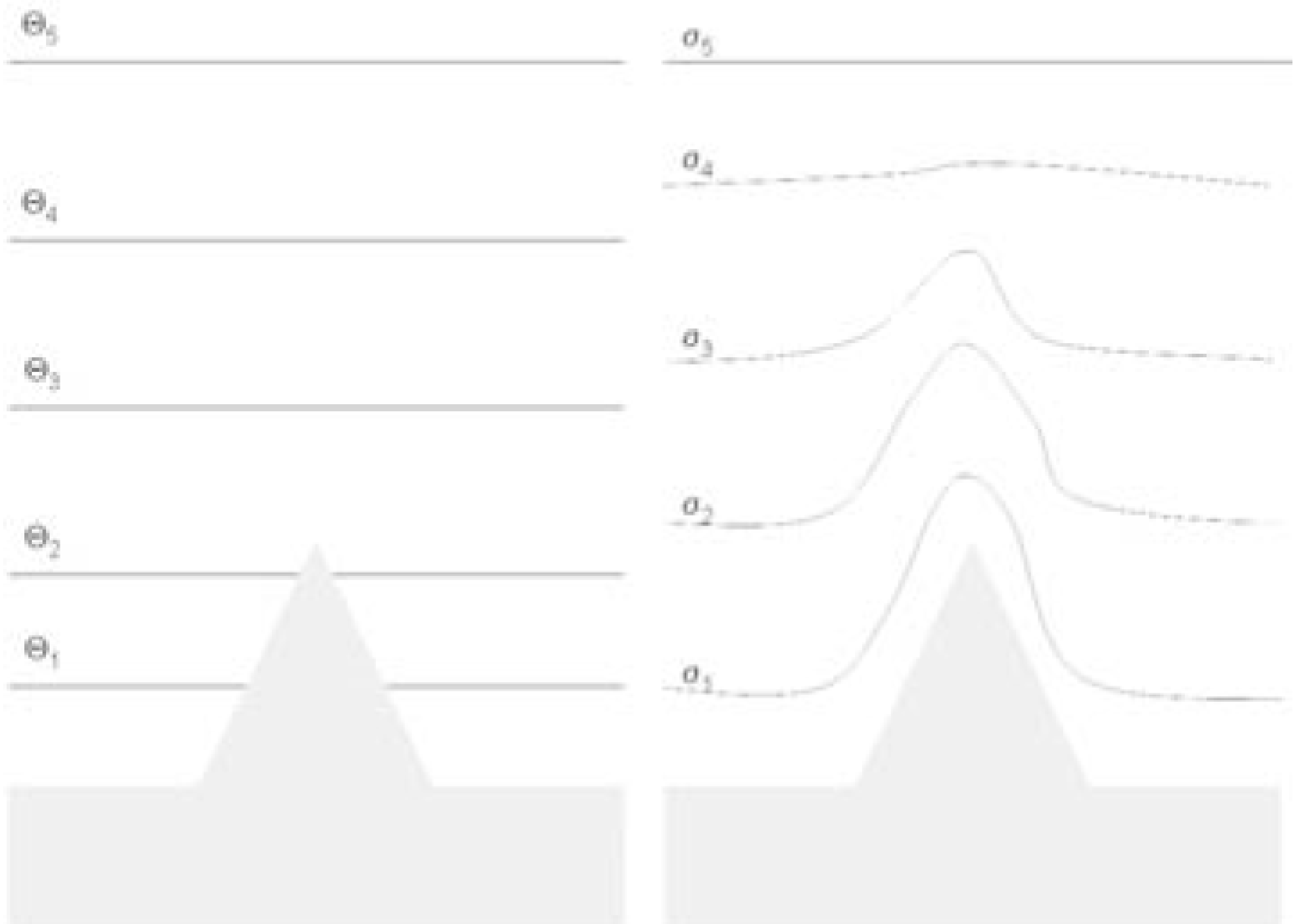
www.atmos.millersville.edu/metall/html/presentations/isen_workshop.ppt

Applications of potential temperature to track inert tracers



From: Mölders et al. 2011

Applications of potential temperature as coordinates



Dry adiabatic lapse rate

$$d\Theta = \frac{\Theta}{T}(dT - \frac{1}{\rho c_p}dp)$$

$$c_p d\Theta = \frac{\Theta}{T}(c_p dT - \frac{1}{\rho}dp) = \frac{\Theta}{T}T ds$$

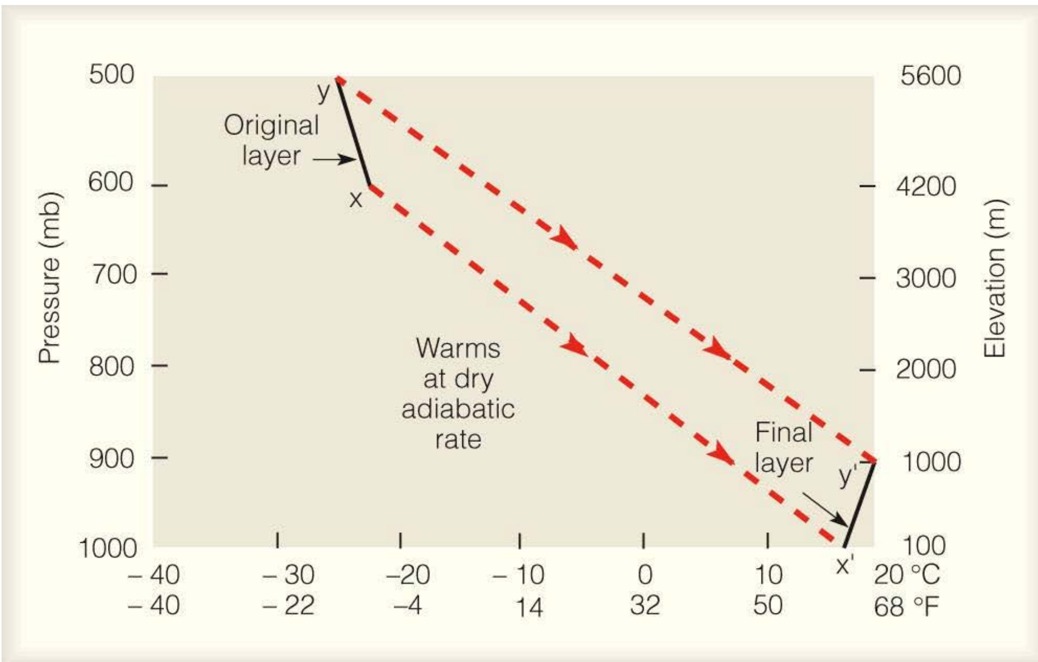
$$c_p \frac{dT}{T} + \frac{g}{T_e}dz = 0$$

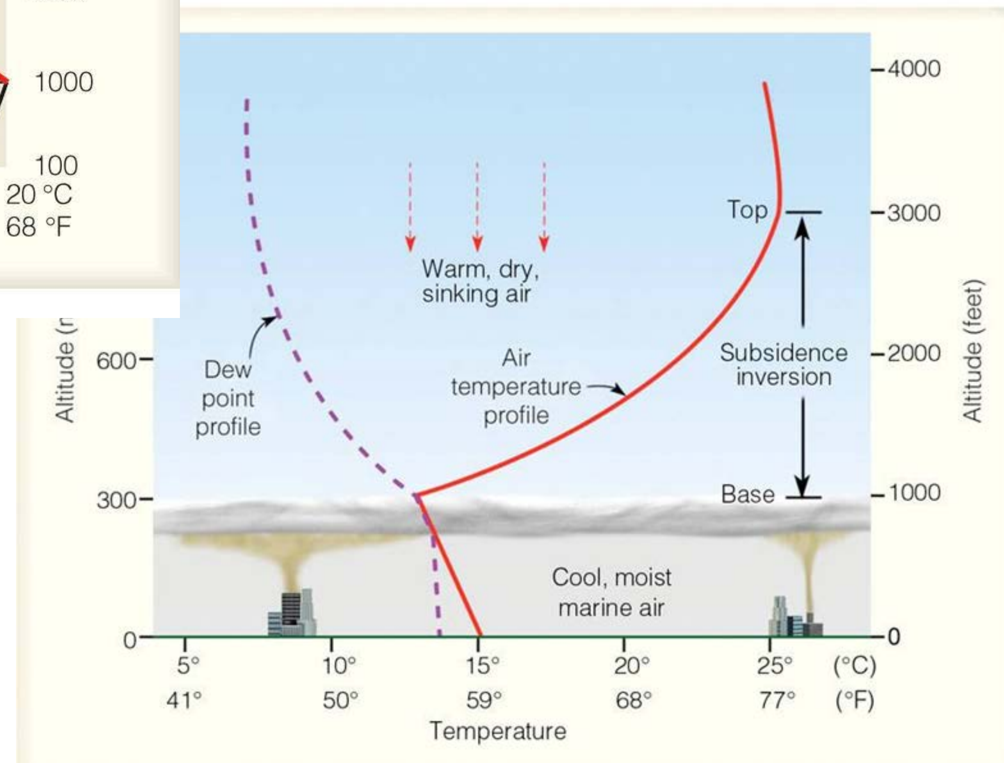
$$\frac{dT}{dz} = -\frac{g}{c_p}\frac{T}{T_e}$$

dp_e=-$r_e$gdz with p_e=$r_e R_d T_e$ and p_e=p
Since T_e~T and g/c_p=0.98K/100m

$$\frac{dT}{dz} =: -\Gamma_d = -\frac{g}{c_p} \approx -0.98\frac{K}{100m}$$

Application of Γ_d for forecasting subsidence inversions





Application of Γ_d to examine the dry static energy

dry static energy:= h+gz   with $h=u+pv=c_pT$

$$\frac{dh}{dz} = -g$$

$d(h+gz)/dz=0$

$d\theta/dz=0$

Diabatic heating

$\delta Q \neq 0 \Rightarrow \theta$ is not conserved $\Rightarrow d\theta/dz \neq 0$

$$dln\Theta - dlnT = -\kappa dlnp$$

$$\frac{\partial T}{\partial z} = \frac{\partial \Theta}{\partial z} - \frac{g}{c_p}$$

Thermodynamic energy eq. $Q \equiv \frac{DS}{Dt} = c_p \frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt}$ under diabatic heating:

$$\frac{D\Theta}{Dt} = \frac{Q}{c_p}\left(\frac{p}{p_0}\right)^{-\kappa}$$