

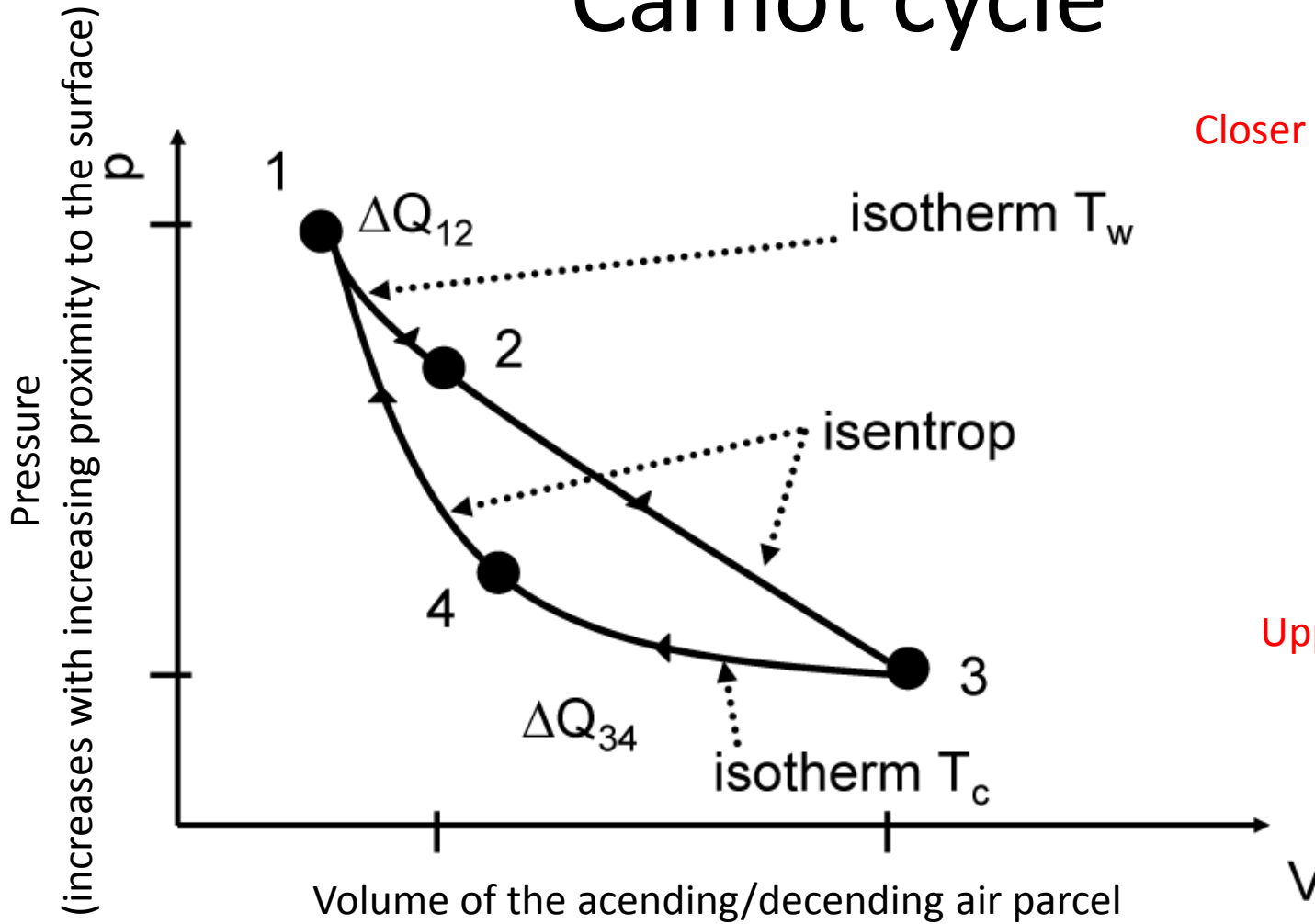
Unit 4



Second Law of thermodynamics

Nicole Mölders

Carnot cycle



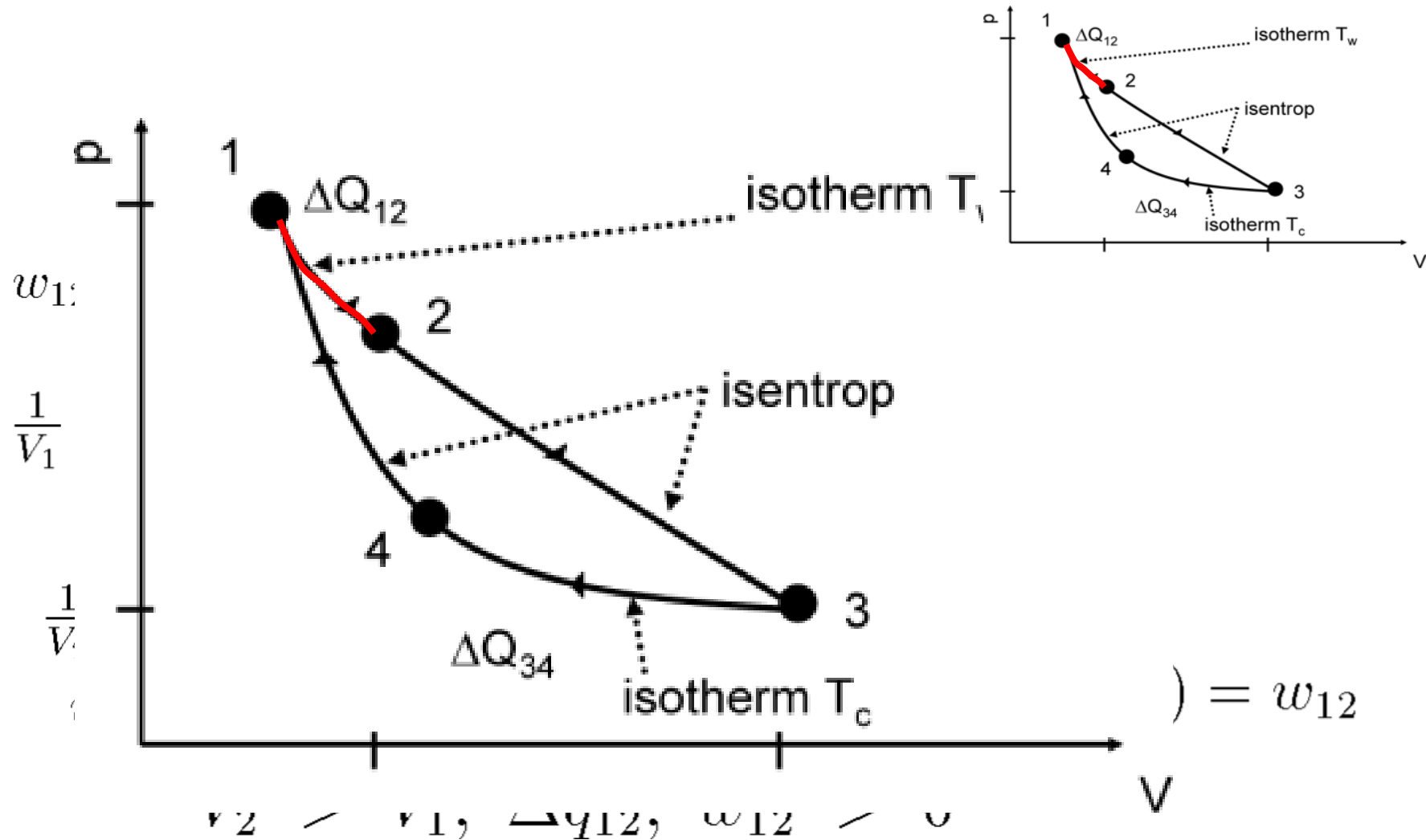
Closer to the Earth's surface

Upper levels in the troposphere

$$\Theta_1 = T_w \left(\frac{1000}{p_1} \right)^{\frac{R_d}{c_p}} = T_c \left(\frac{1000}{p_4} \right)^{\frac{R_d}{c_p}}$$

$$\Theta_2 = T_c \left(\frac{1000}{p_3} \right)^{\frac{R_d}{c_p}} = T_w \left(\frac{1000}{p_2} \right)^{\frac{R_d}{c_p}}$$

1 → 2: Reversible isothermal expansion



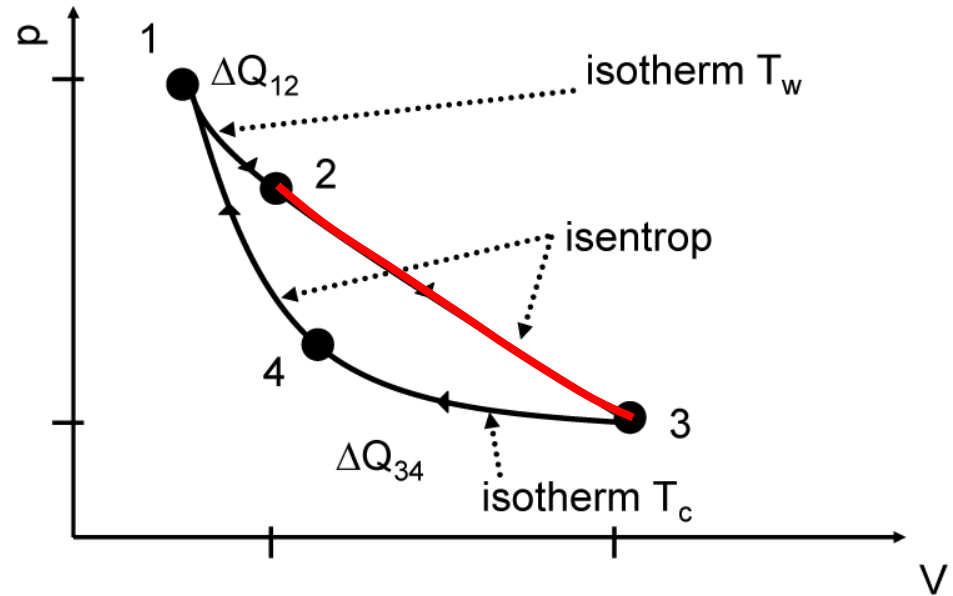
THM: heat can be added without change in T!

2 → 3: Reversible adiabatic expansion

$$\Delta q_{23} = 0$$

$$\Delta u_{23} = -c_v(T_w - T_c)$$

$$w_{23} = -\Delta u_{23} = c_v(T_w - T_c)$$



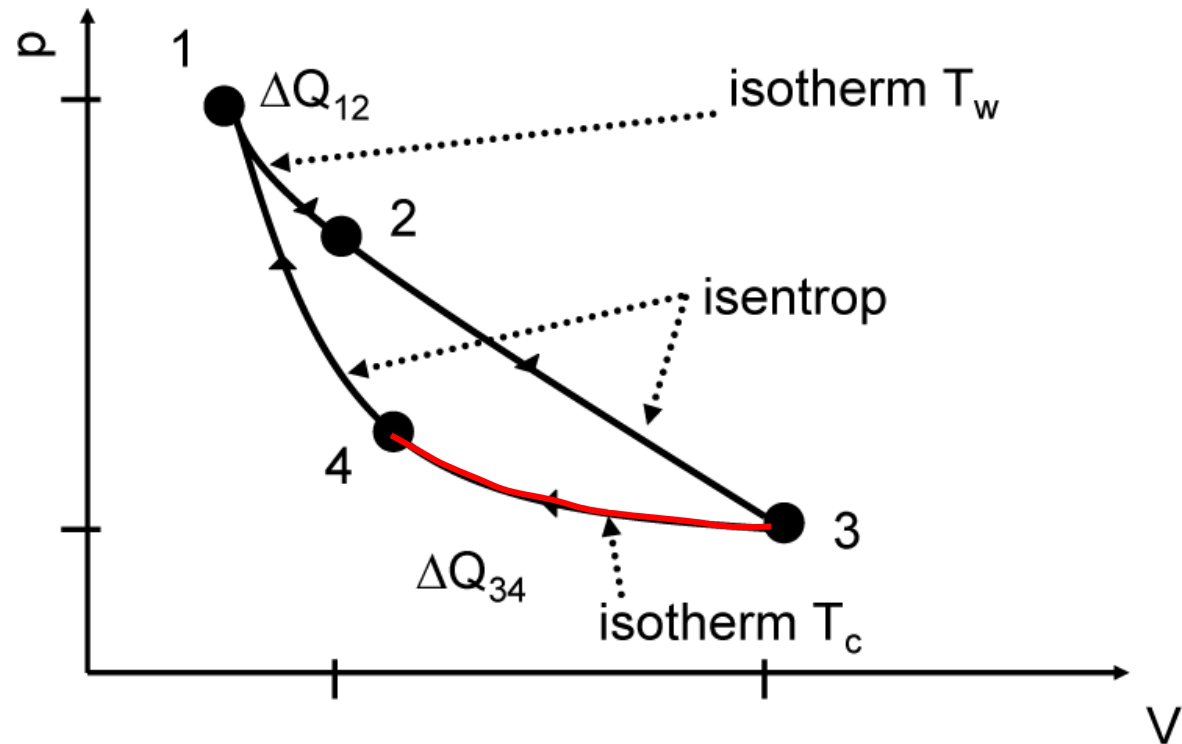
3 → 4: Reversible isothermal compression

$$\Delta u_{34} = 0$$

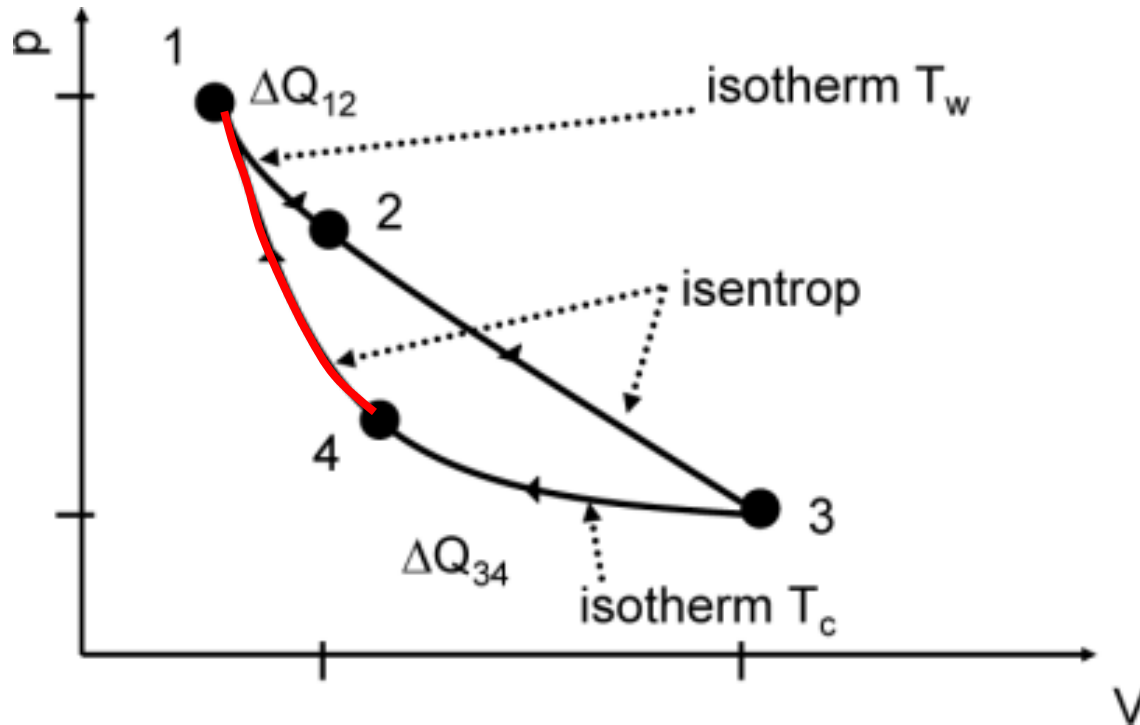
$$w_{34} = -R_d T_c \ln \frac{V_3}{V_4} = -R_d T_c \ln \left(\frac{p_1}{p_3} \left(\frac{T_c}{T_w} \right)^{\frac{c_p}{R_d}} \right)$$

$$q_{34} = -c_p T_c \left(\frac{\Theta_2}{\Theta_1} \right)$$

$$q_{34} = W_{34}$$



4 → 1: Reversible adiabatic compression



$$q_{41} = 0$$

$$\Delta u_{41} = c_v(T_c - T_w) > 0$$

$$w_{41} = -\Delta u_{41} = -c_v(T_w - T_c) < 0$$

Carnot cycle - THM

$$w = w_{12} + w_{23} + w_{34} + w_{41}$$
$$q = q_{12} + q_{23} + q_{34} + q_{41}$$

$$q = c_p(T_w - T_c) \ln\left(\frac{\Theta_2}{\Theta_1}\right)$$

$$w = q_{12} + q_{34} \text{ and } q_{34} < 0$$

$$w = R_d(T_w - T_c) \ln\left(\frac{p_1/p_3(T_c/T_w)}{c_p/R_d}\right)$$

$$\eta = \frac{\Delta q_{12} + \Delta q_{34}}{\Delta q_{12}} = 1 + \frac{\Delta q_{34}}{\Delta q_{12}} = 1 - \frac{T_c}{T_w}$$

THM: Carnot-cycle has maximum possible efficiency of all processes between T_w and T_c

Thermodynamics potentials

Potential	Variables
$U(S, V, N)$	S, V, N
$H(S, P, N)$	S, P, N
$F(T, V, N)$	V, T, N
$G(T, P, N)$	P, T, N

$$dU = T dS - P dV + \mu dN$$

$$dH = T dS + V dP + \mu dN$$

$$dF = -S dT - P dV + \mu dN$$

$$dG = -S dT + V dP + \mu dN$$

Maxwell relation: $\left(\frac{\partial T}{\partial V}\right)_{S, N} = -\left(\frac{\partial P}{\partial S}\right)_{V, N}, \dots$

Example: isolated air parcel

$$dU(S, V, N) = TdS - PdV + \mu dN$$

$$dU(S, V, N) = \left(\frac{\partial U}{\partial S} \right)_{V, N} dS + \left(\frac{\partial U}{\partial V} \right)_{S, N} dV + \left(\frac{\partial U}{\partial N} \right)_{S, V} dN$$

$$\left(\frac{\partial U}{\partial S} \right)_{V, N} = T \quad \left(\frac{\partial U}{\partial V} \right)_{S, N} = -P \quad \left(\frac{\partial U}{\partial N} \right)_{S, V} = \mu$$