Unit 6

Phase transitions

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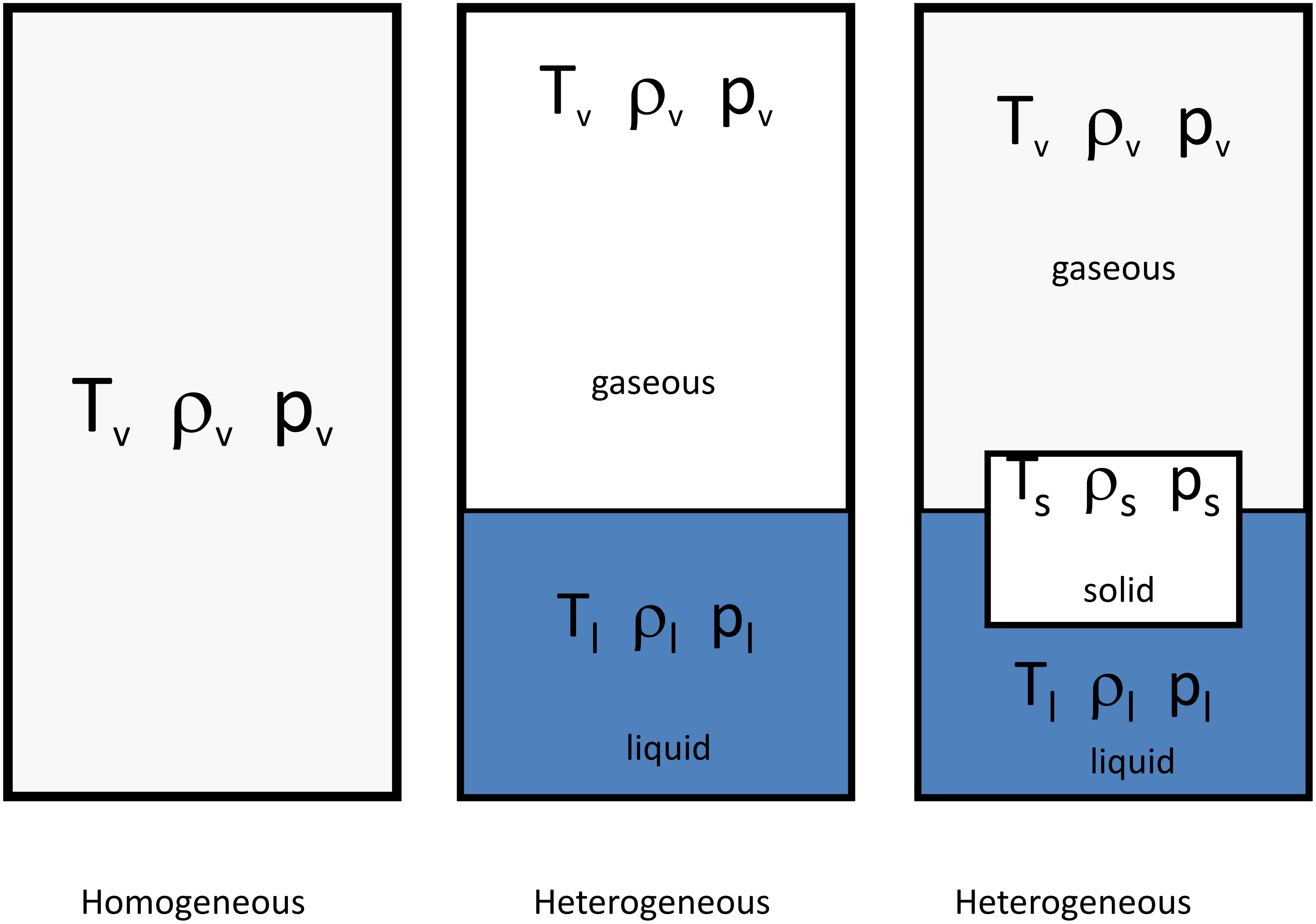
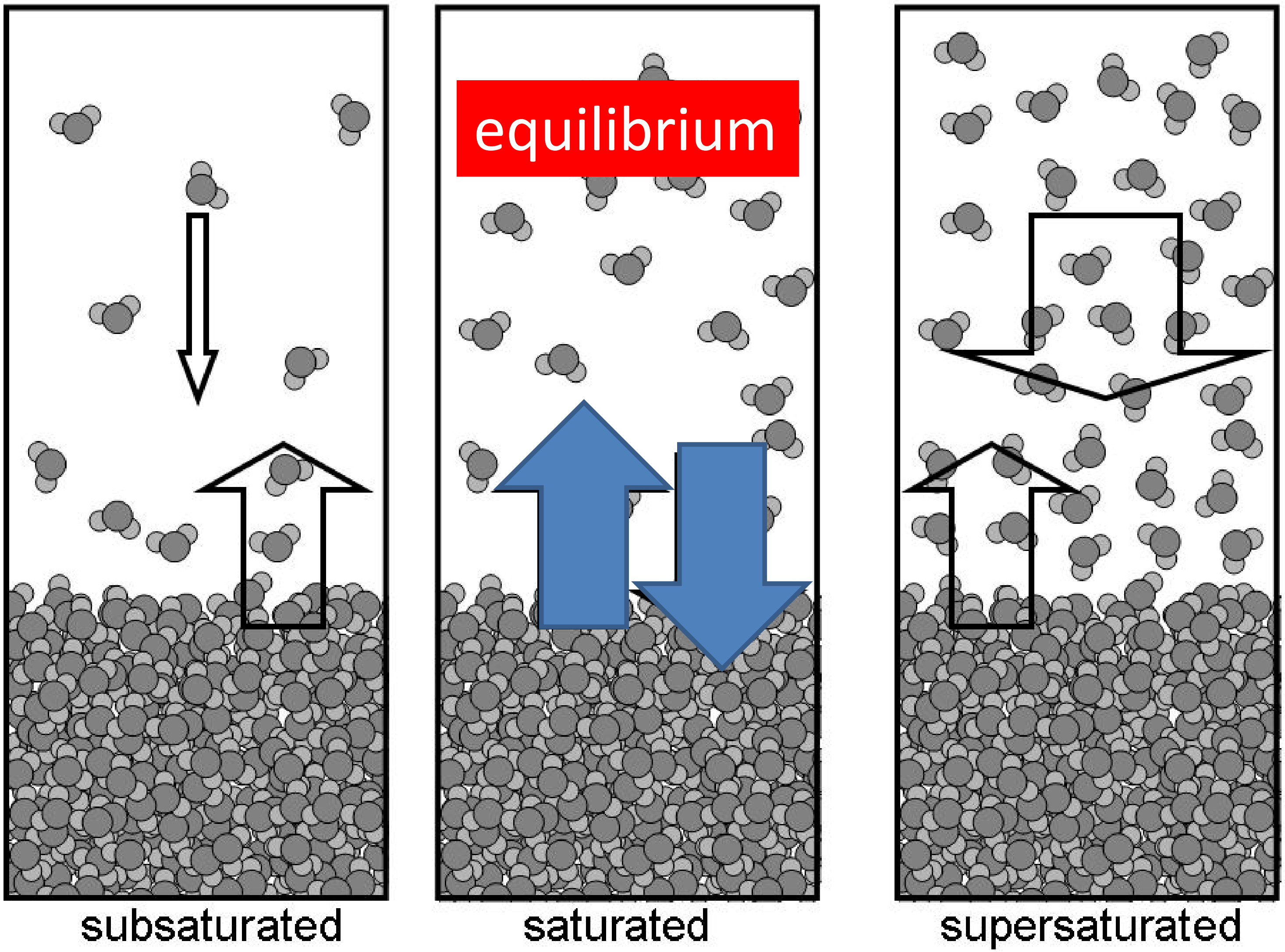
Heterogeneous system



Phase transitions

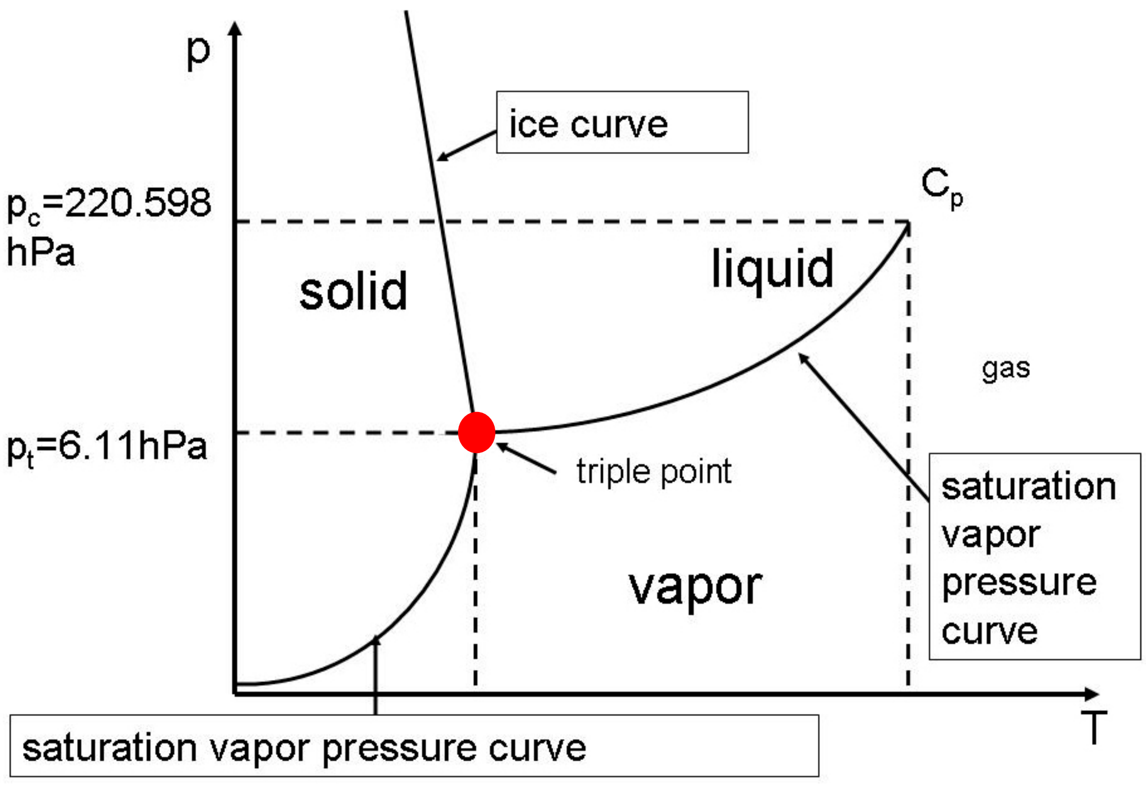


Thermodynamic properties of water

$\Rightarrow \mu = \mu\ (p,T) = \mu'\ (p',T')$

$\Rightarrow$ p=f(T) or T=f(p)

$\Rightarrow$ In one-component system with 2 phases, 1 independent variable exists!



Latent heat

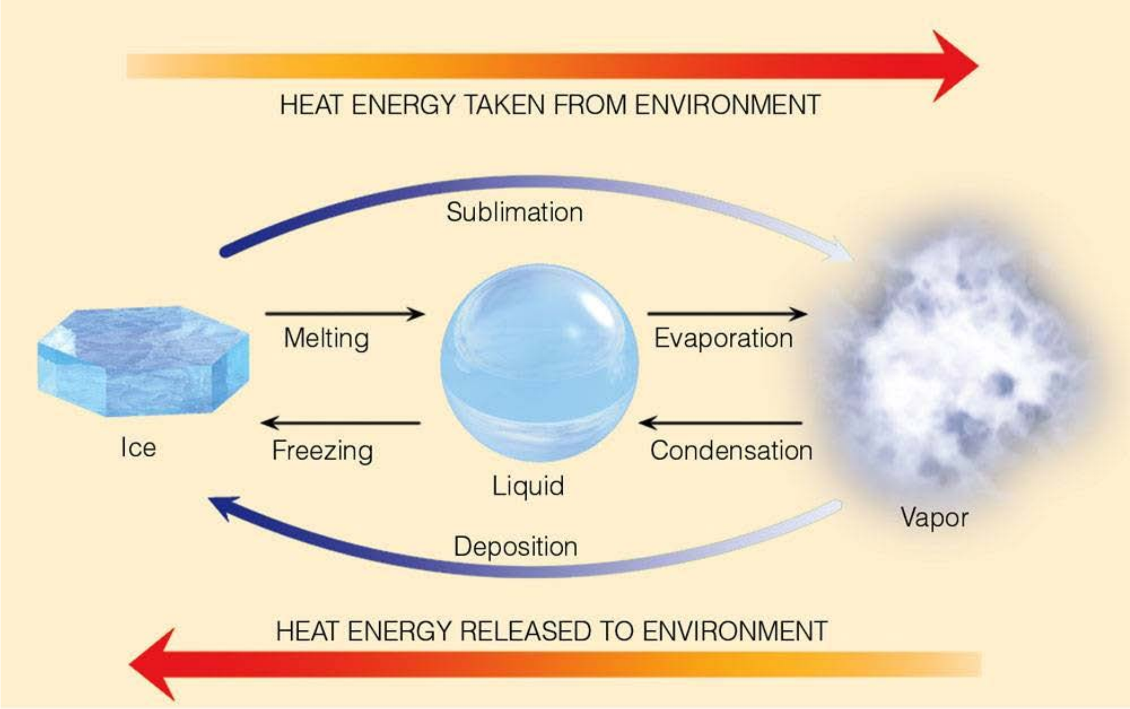
$$L_v = H_v - H_w = U_v - U_w + p_{wv}(V_v - V_w)$$
$$L_f = H_w - H_i = U_w - U_i + p_{wi}(V_w - V_i)$$
$$L_s = H_v - H_i = U_v - U_i + p_{vi}(V_v - V_i)$$

$dU=L-pdV=L-R_vT$

$dV=V_v-V_{wi}\sim V_v$ for vaporization & sublimation

$dV\sim 0$ for fusion

$$dU = L$$
$$dU = L - mR_vT$$



Clausius-Clapeyron-equation

$$V = m_w \alpha_w + m_v \alpha_v \qquad U = m_w u_w + m_v u_v$$

$$V + dV = (m_w - dm)\alpha_w + (m_v + dm)\alpha_v \text{ or } dV = (\alpha_v - \alpha_w)dm$$

$$dU = (u_v - u_w)dm. \qquad u_v - u_w + e_{sw}(\alpha_v - \alpha_w) = l_v$$

$$\frac{dU}{dV} = \frac{l_v}{\alpha_v - \alpha_w} - e_{sw}$$

Joule's law

TdS=dU+pdV

$$T \frac{\partial p}{\partial T} |_V = \frac{\partial U}{\partial V} |_T + p. \qquad \frac{\partial U}{\partial V} |_T = 0$$

$$\frac{\partial U}{\partial V} |_T = T \frac{\partial p}{\partial T} |_V - p$$

$$\frac{\partial U}{\partial V} |_T = \frac{dU}{dV} \text{ or } \frac{\partial e_{sw}}{\partial T} |_V = \frac{de_{sw}}{dT}$$

$$\boxed{\frac{de_{sw}}{dT} = \frac{l_v}{T\Delta v}}$$

Analystical solution of the Clausius-Clapeyron-equation

l_v=const. $v_w << v_v$ → $\Delta v \sim v_v$

$$\frac{de_{sw}}{dT} = \frac{l_v}{T}\frac{e_{sw}}{R_v T}$$

$$\frac{de_{sw}}{e_{sw}} = \frac{l_v}{R_v}\frac{dT}{T^2}$$

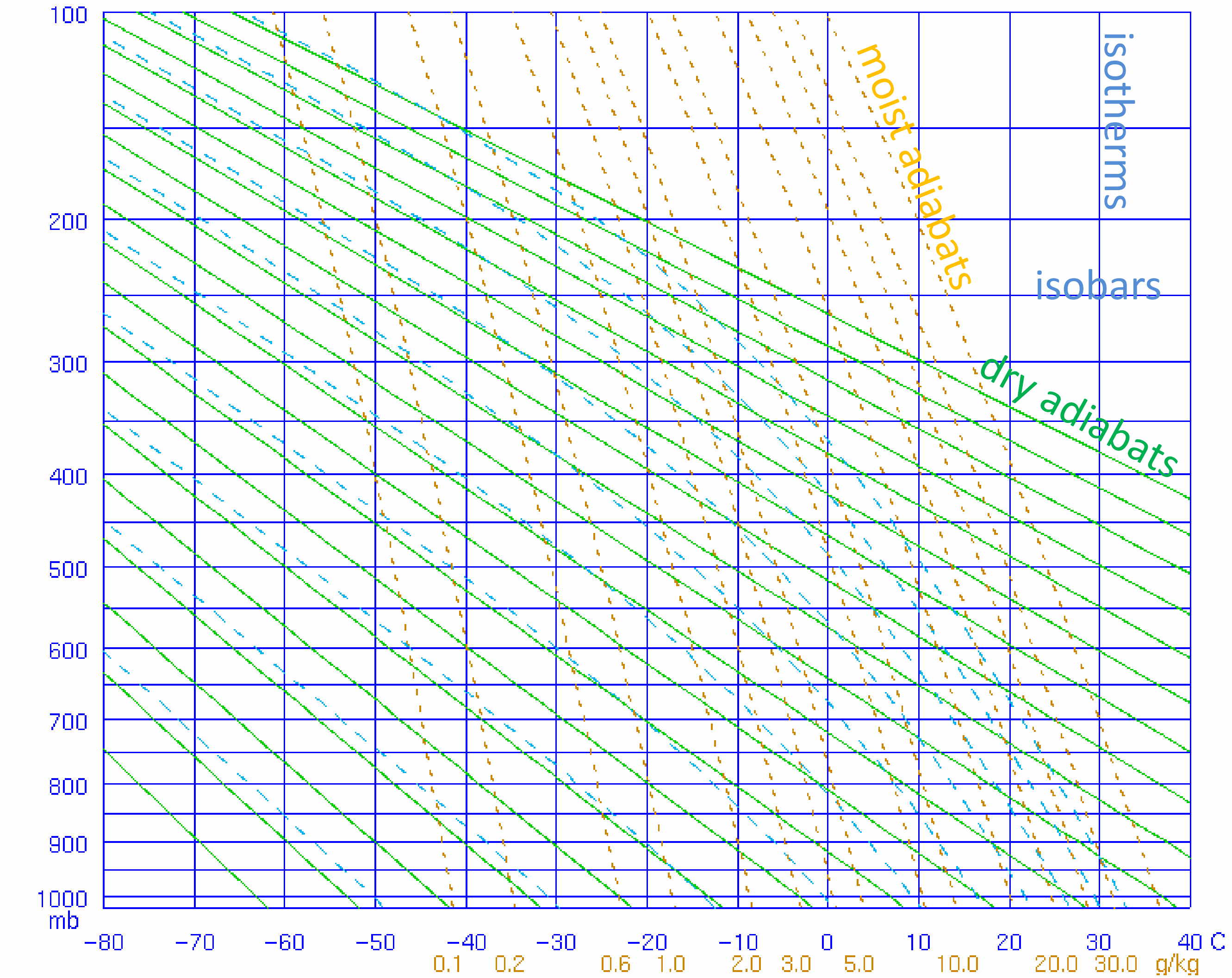
$$ln\frac{e_{sw}}{e_{sw,o}} = -\frac{l_v}{R_v}\left(\frac{1}{T} - \frac{1}{T_o}\right) = \frac{l_v}{R_v T_o}\left(\frac{T-T_o}{T}\right)$$

$$e_{sw} = e_{sw,o} exp\left(\frac{l_v}{R_v T_o}\frac{T - T_o}{T}\right)$$

With $l_v=2.5 * 10^6$J/kg, R_v=461.5 J/(kg K), $e_{sw,o}$=6.1078hPa, T_o=273.15K

$\Rightarrow l_v/(R_v T_o)=19.81$

Stüve diagram



http://en.wikipedia.org/wiki/File:Stuve-diagram.gif

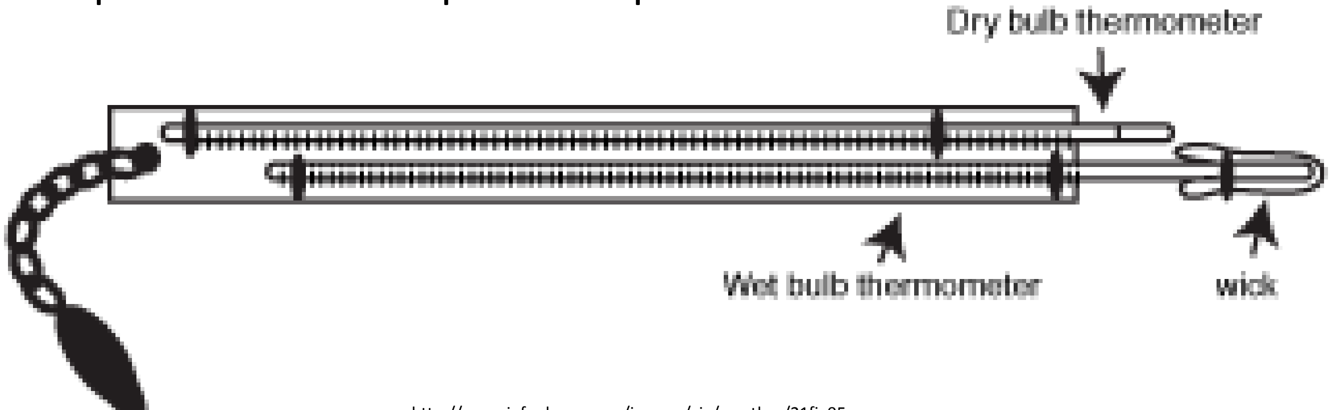
Relative humidity is a relative measure for the degree of saturation

$$rh = \frac{e}{e_s} = \frac{q}{q_s} = \frac{r}{r_s}$$

Hair hygrometer measures relative humidity

http://en.wikipedia.org/wiki/Hygrometer#mediaviewer/File:Haar-Hygrometer.jpg

Sling psychrometer for measuring humidity by use of Temperature and dewpoint temperature

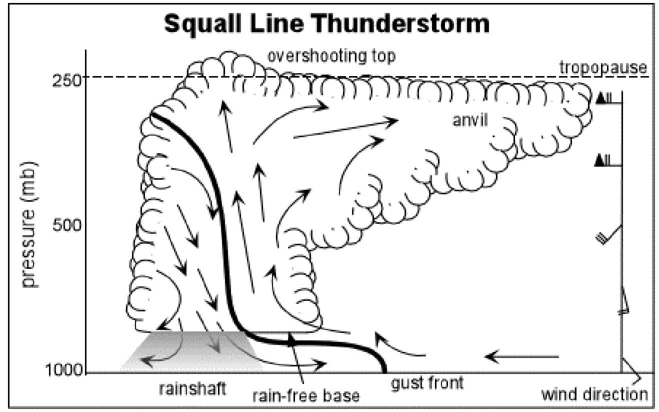


http://www.infoplease.com/images/cig/weather/21fig05.png

Dew-point temperature gradient

$$\frac{dp}{dz} = -\frac{p}{R_d T} g$$

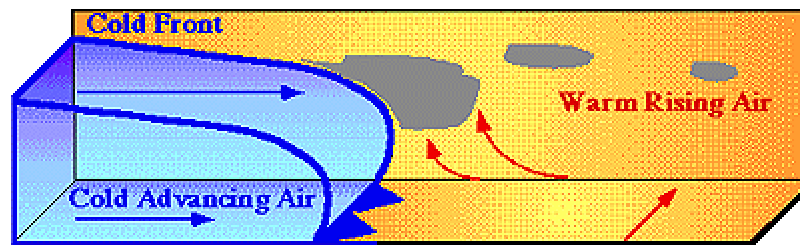
$$\boxed{\frac{1}{e_s}\frac{de_s}{dT} = \frac{L_v}{R_v T^2}}$$



http://ww2010.atmos.uiuc.edu/guides/mtr/svr/modl/line/gifs/squall3.gif

$$\frac{de_s}{dz} = \frac{de_s}{dT}\frac{dT}{dz}$$

$$\frac{1}{e_s}\frac{de_s}{dz} = \frac{L_v}{R_v T^2}\Gamma_d$$



http://ww2010.atmos.uiuc.edu/guides/mtr/af/frnts/gifs/home.gif

$$\frac{1}{e}\frac{de}{dz} = \frac{1}{p}\frac{dp}{dz}$$

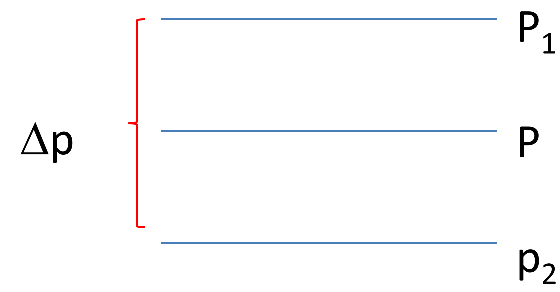
$$\boxed{\frac{dT_d}{dz} = \frac{de}{dz}\left(\frac{de}{dT_d}\right)^{-1} = \frac{1}{e}\frac{de}{dz}\left(\frac{1}{e_s(T_d)}\frac{de_s(T_d)}{dT_d}\right)^{-1} = \frac{0.172K}{100m}}$$

Vertical mixing requires to know your extensive and intensive quantities for correct calculations

$$T \approx \frac{m_1 T_1' + m_2 T_2'}{m}$$

$$q \approx \frac{m_1 q_1' + m_2 q_2'}{m}$$

$$\Theta \approx \frac{m_1 \Theta_1' + m_2 \Theta_2'}{m}$$



$$\overline{x} = \frac{\int_0^m x\,dm}{m}$$

$$\overline{x} = \frac{\int_0^z x\,dz}{\int_0^z \rho\,dz} = -\frac{\int_{p_1}^{p_2} x\,dp}{p_1 - p_2}$$

h http://www.susanstevenson.com/Journal/2013/November/90210PushingCarOffCurbP.jpg