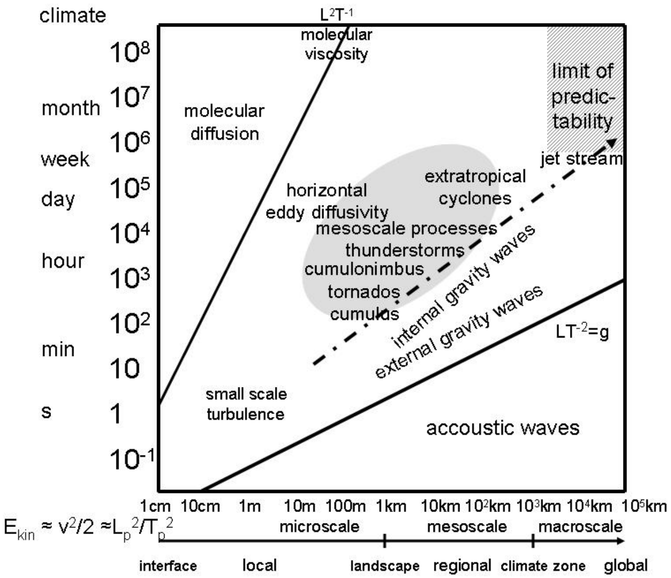
Unit 19

Scale analysis and balanced flows

Nicole Mölders

Scale analysis



Example: mid-latitudes, synoptic scale

Horizontal velocity (U)	≈ 10 m s^{-1}	(u,v)
Vertical velocity (W)	≈ 10^{-2} m s^{-1}	(w)
Horizontal Length (L)	≈ 10^{6} m	(dx,dy)
Vertical Height (H)	≈ 10^{4} m	(dz)
Angular Velocity (Ω)	≈ 10^{-4} s^{-1}	(Ω)
Time Scale (T)	≈ 10^{5} s	(dt)
Frictional Acceleration	≈ 10^{-3} m s^{-2}	(F_{rx}, F_{ry}, F_{rz})
Gravitational acceleration	≈ 10 m s^{-2}	(g)
Horizontal Pressure Gradient (Δp)	≈ 10^{3} Pa	(dp/dx, dp/dy)
Vertical Pressure Gradient	≈ 10^{5} Pa	(dp/dz)
Air Density (ρ)	≈ 1 m^{3} kg^{-1}	(ρ)
Coriolis Effect (f)	≈ 1	(2sinφ, 2cosφ)

⇒ Using these values in the governing equations, numerous terms can be neglected…..

Hydrostatic approximation derived from scale analysis

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + g + \frac{1}{\rho}\frac{\partial p}{\partial z} = 0$$

$$\frac{\partial w}{\partial t} \approx \frac{W}{T} \approx \frac{10^{-2} m/s}{10^5 s} \approx 10^{-7} m s^{-2}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} \approx \frac{UW}{L} \approx \frac{10^{-1} m^2/s^2}{10^6 m} \approx 10^{-7} m s^{-2}$$

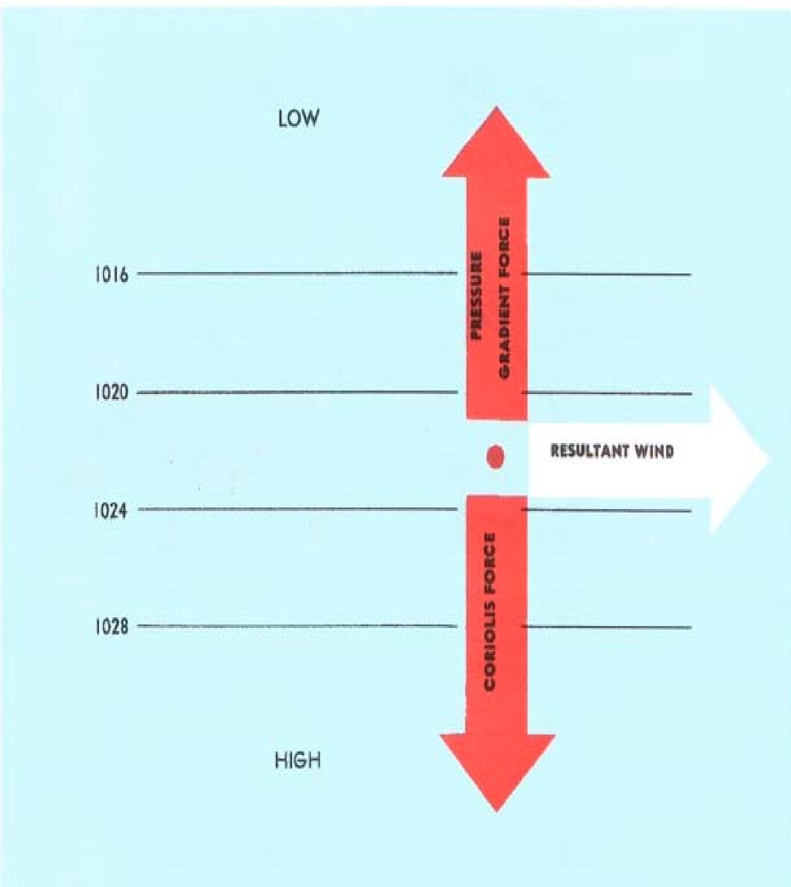
$$w\frac{\partial w}{\partial z} \approx \frac{W^2}{H} \approx \frac{10^{-4} m^2/s^2}{10^4 m} \approx 10^{-8} m s^{-2}$$

$$\frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \approx 10^{-7} m s^{-2}$$

$$\frac{\partial p}{\partial z} = -\rho g$$

Geostrophic wind approximation from scale analysis

- Scale analysis on Euler eq. for synoptic scale flow



$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} \approx \frac{U}{T} \approx \frac{10^1 m/s}{10^5 s} \approx 10^{-4} ms^{-2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \approx \frac{U^2}{L} \approx \frac{10^2 m^2/s^2}{10^6 m} \approx 10^{-4} ms^{-2}$$

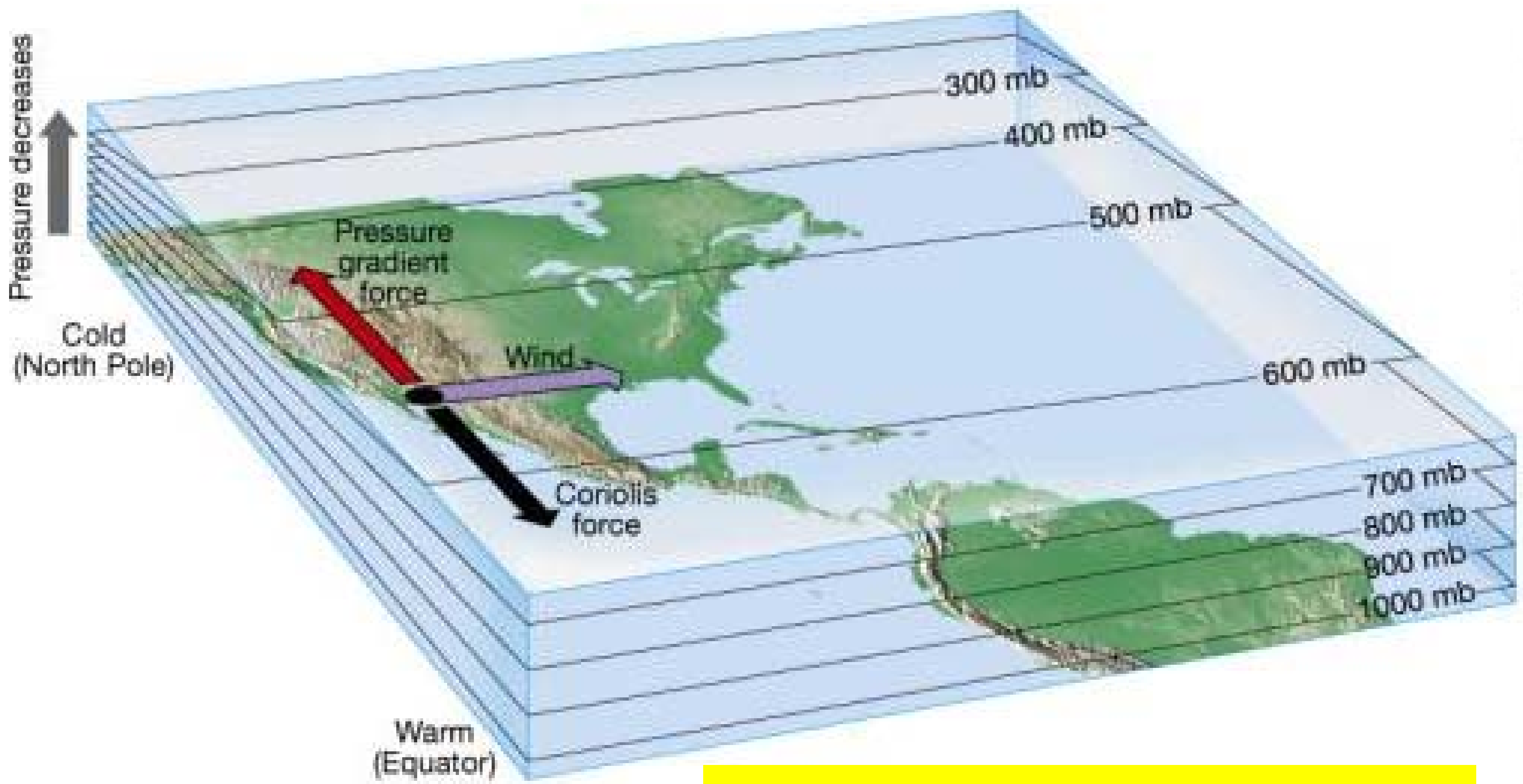
$$w\frac{\partial u}{\partial z} \approx \frac{UW}{L} \approx \frac{10^{-1} m^2/s^2}{10^4 m} \approx 10^{-5} ms^{-2}$$

$$\frac{Du}{Dt} \approx \frac{Dv}{Dt} \approx 10^{-4} ms^{-2}$$

$$fV \approx fU \approx 10^{-4} s^{-1} 10 ms^{-1} \approx 10^{-3} ms^{-2}$$

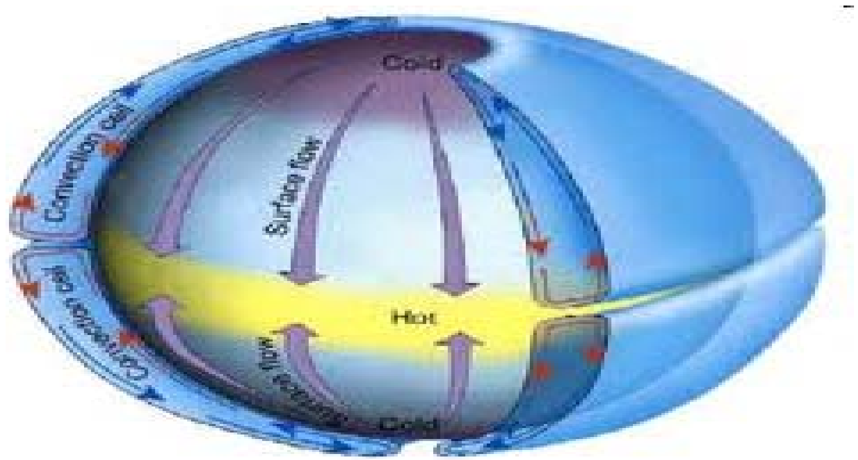
$$fv = \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad -fu = \frac{1}{\rho}\frac{\partial p}{\partial y}$$

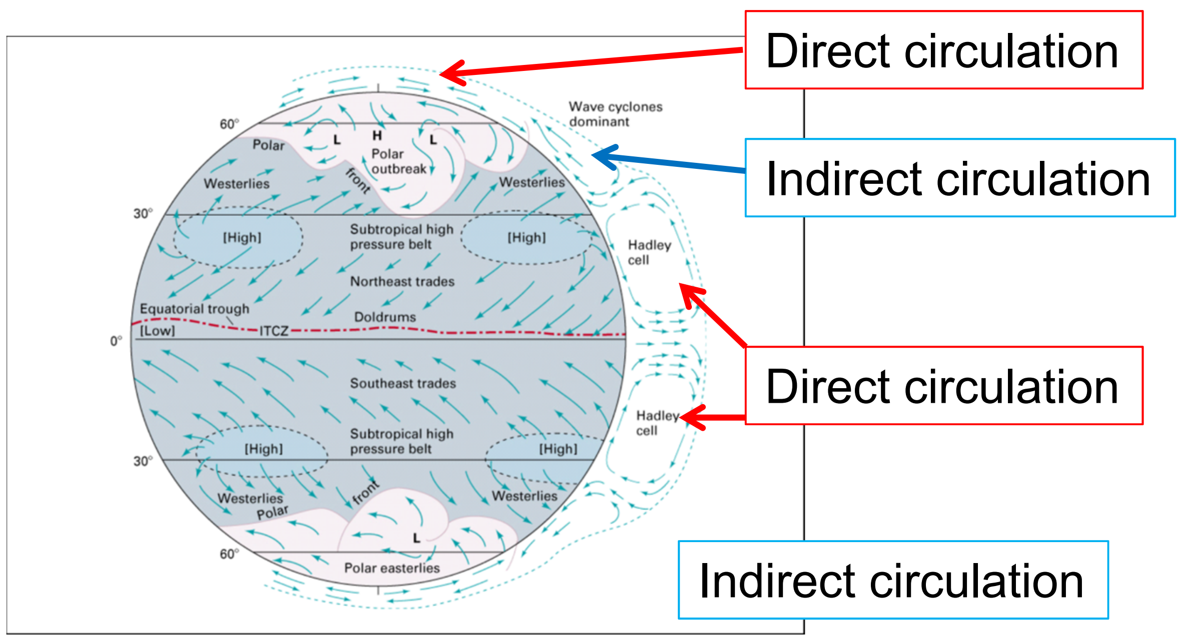
Geostrophic wind blows with downslope isobars to the left in NH



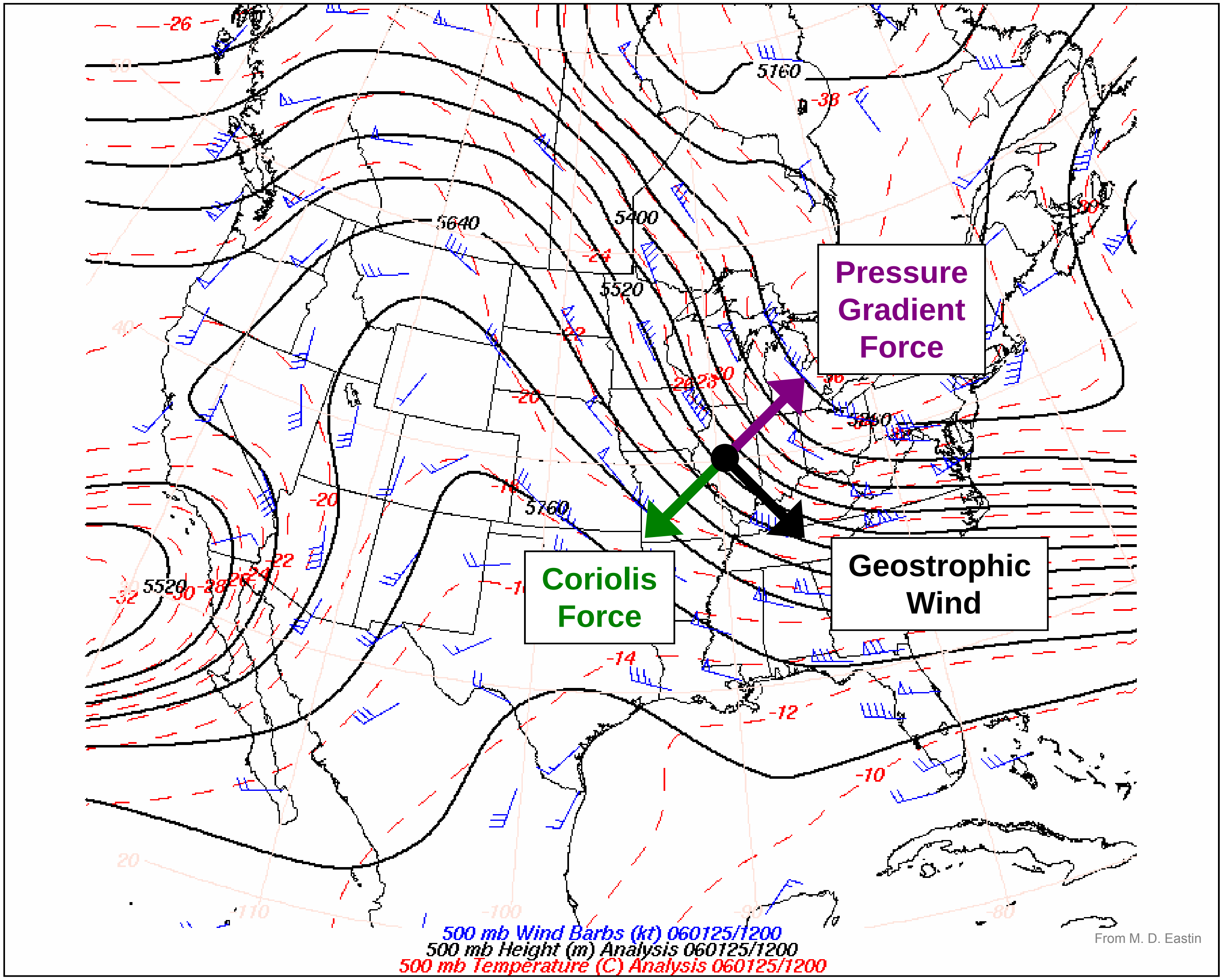
Synoptic rule of thumb: with your back to the wind the low is to your left

Interaction dynamics-thermodynamics

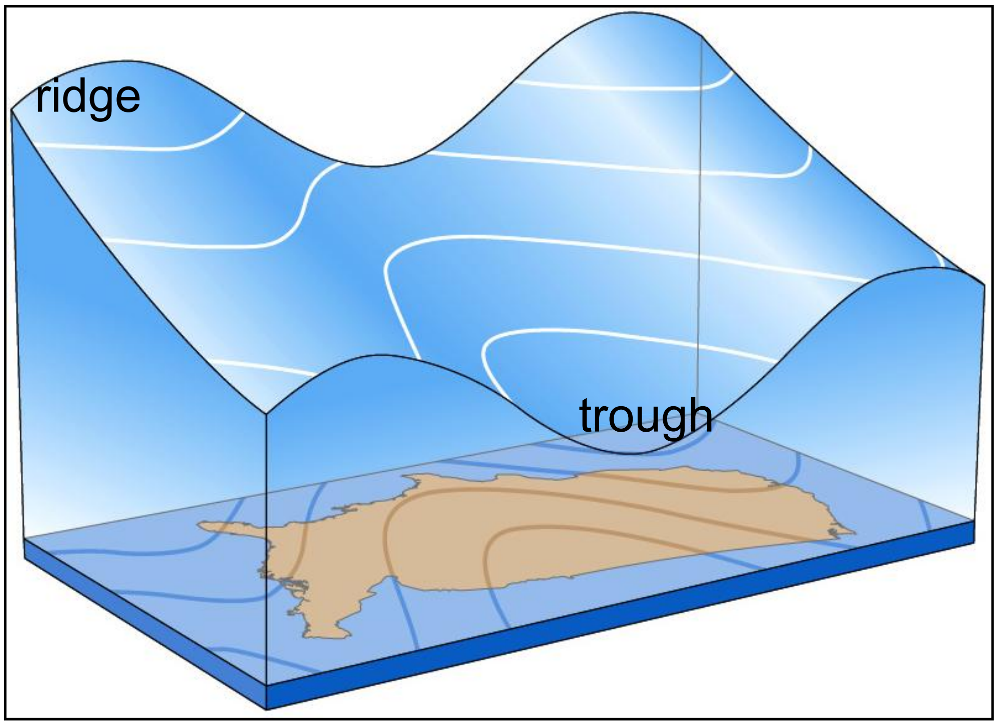




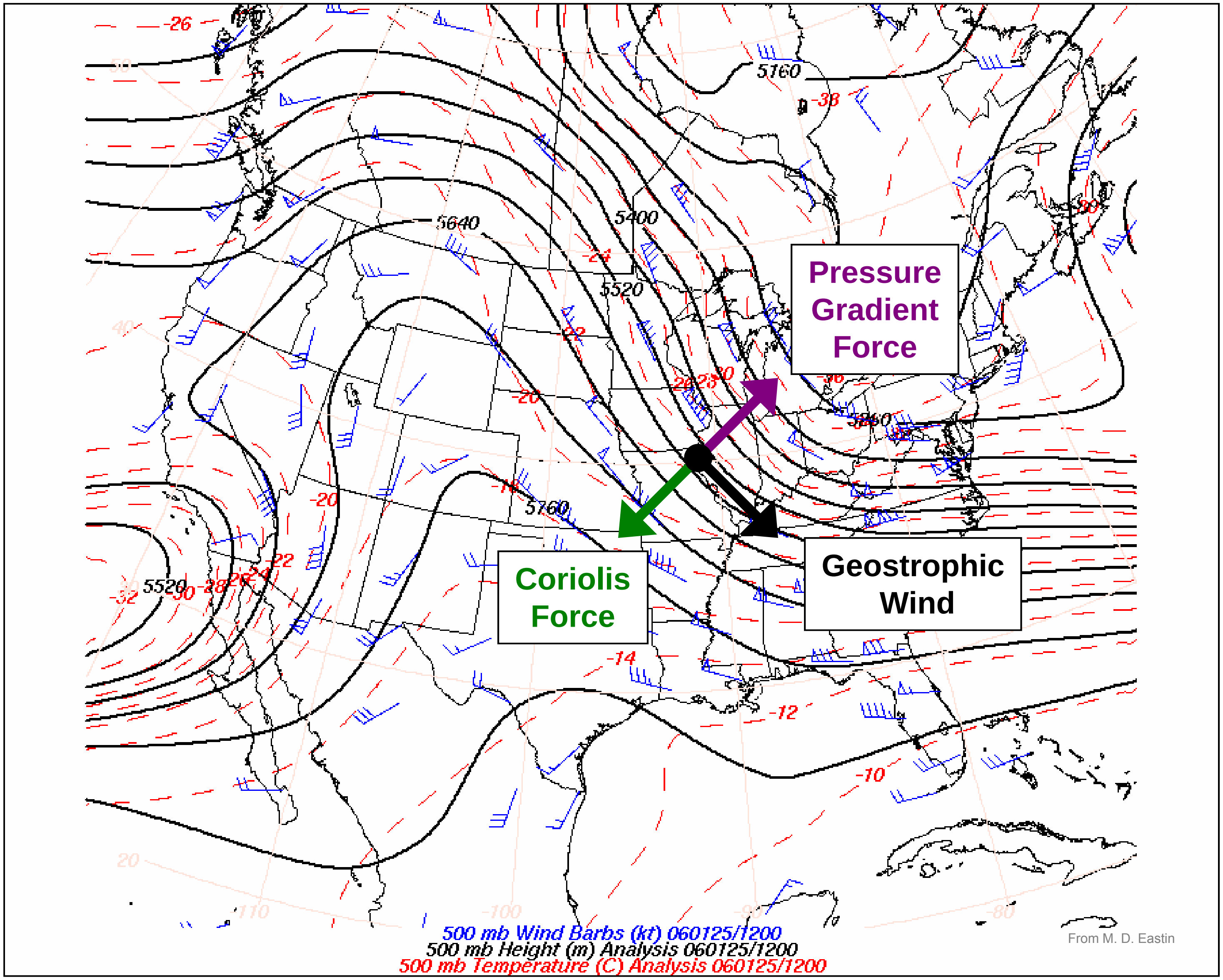
Geostrophic balance is valid only for straight flow



3D illustration of a trough and ridge



Geostrophic balance is valid only for straight flow

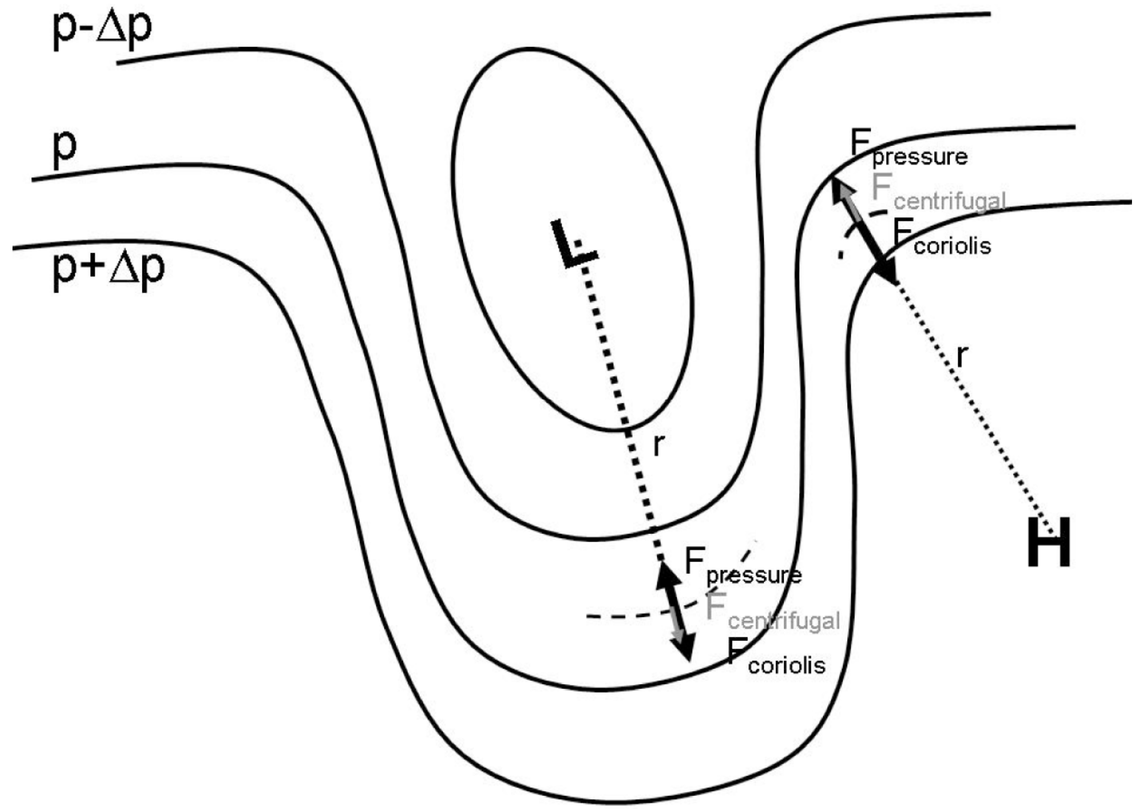


$$R_o \equiv \frac{U^2/L}{f_o U} = \frac{U}{(f_o L)}$$

Gradient wind balance

In natural coordinates

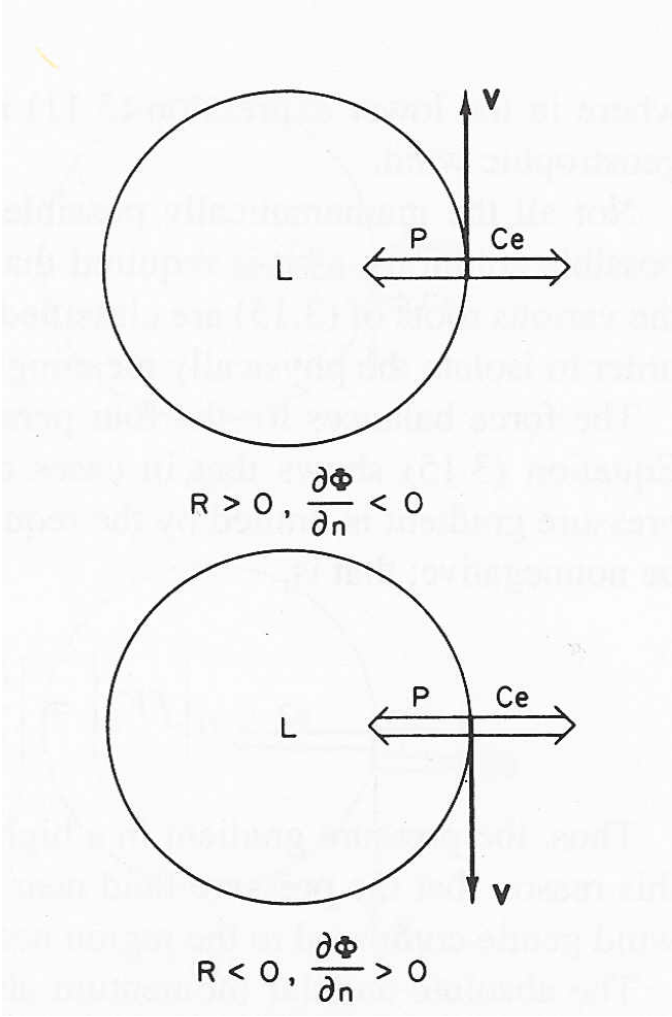
$$\frac{v^2}{r} + fv = \frac{1}{\rho}\frac{dp}{dr}$$



Cyclostrophic motion

$$\frac{v^2}{r} + \frac{1}{\rho}\frac{dp}{dr} = 0$$

$$\frac{V^2}{R} = -\frac{\partial \Phi}{\partial n}$$



Antitriptic wind