

## Unit 1 ATM401, ATM601 and CHEM601

### Application, analysis, and evaluation

1. **All students:** As discussed in Lectures in Meteorology, hydrogen escape has been important in the evolution of the Earth's atmosphere. In the 1920s, James Jeans presented the theory of the thermal escape flux of molecules from the top of the atmosphere given as

$$F = \frac{n u}{2\pi^{0.5}} \exp\left(-\frac{v^2}{u^2}\right) \left(\frac{v^2}{u^2} + 1\right)$$

Here  $n$ ,  $v$ , and  $u$  are the number density of molecules of a given species at the exobase (the escape level), the escape velocity for the Earth, and the most probable molecular velocity, respectively. The latter is given by kinetic theory as

$$u = \sqrt{\frac{2kT}{m}}$$

where  $m$  and  $k = 1.38066 \cdot 10^{-23} \text{ J K}^{-1}$  are the molecular mass of the species and the Boltzmann constant. The escape velocity of the Earth can be derived from the potential energy  $E_{pot} = g m R$  for an object of mass  $m$  at the distance of the center of the Earth of mass  $M = 5.97 \cdot 10^{24} \text{ kg}$  and the kinetic energy of the object  $E_{kin} = \frac{1}{2} m v^2$  with the velocity  $v$ . The escape velocity is the exact velocity  $v$  needed to escape from the energy source.

Determine the probable escape velocity for a hydrogen atom ( $m = 1.67 \cdot 10^{-27} \text{ kg}$ ) and an oxygen atom ( $m = 2.626778 \cdot 10^{-26} \text{ kg}$ ) at  $500 \text{ km}$  height assuming a moderately active sun. Which assumption do you have to make about the temperature? Note you don't need the first equation for this part of the problem. Determine the escape velocity for an obstacle in the Earth's atmosphere. Which quantity do you have to look up? Discuss the differences under the aspect of the fate of the atmosphere.

Mercury (Mars) has a gravitational acceleration of  $3.8 \text{ m s}^{-2}$  ( $3.72 \text{ m s}^{-2}$ ) and a radius of about  $2400 \text{ km}$  ( $3398 \text{ km}$ ). Determine the escape velocity for Mercury (Mars) and compare it with the escape velocity (in  $\text{m s}^{-1}$ ) on the Earth.

**ATM601, CHEM601 students:** Show that the escape velocity is independent of the obstacle's mass. Discuss your results.

2. **All students:** Estimate the mean atmospheric pressure averaged over the globe assuming a total mass of the atmosphere of  $m = 5.14 \cdot 10^{18} \text{ kg}$ , a mean radius of the Earth of  $R = 6.37 \cdot 10^6 \text{ m}$  and an acceleration of gravity of  $g = 9.81 \text{ m s}^{-2}$ .

**ATM601, CHEM601 students:** Explain why your result doesn't match the value of the mean sea-level pressure of  $1013.25 \text{ hPa}$ .