Unit 1 ATM401, ATM601 and CHEM601

Solutions

1. Given: $k = 1.38066 \cdot 10^{-23} J\,K^{-1}$ Boltzmann constant
   $M = 5.97 \cdot 10^{24} kg$ mass of Earth
   $m(H) = 1.67 \cdot 10^{-27} kg$
   $m(O) = 2.626778 \cdot 10^{-26} kg$
   $h = 500\,km$ height location of the atoms
   $g(Mercury) = 3.8\,m\,s^{-2}$, $R(Mercury) = 2400\,km = 2400000\,m$
   $g(Mars) = 3.72\,m\,s^{-2}$, $R(Mars) = 3398\,km = 3398000\,m$
   $g(Earth) = 9.81\,m\,s^{-2}$

   $u = \sqrt{\frac{2k\,T}{m}}$

   Temperature was not given in the problem. Lectures in Meteorology discussed typical values for a quite and active Sun as 600 K and ¿1000 K at 500 km, respectively. Thus, assume something in between, for instance, 700K.

   $u(H) = \sqrt{\frac{2 \cdot 1.38 \cdot 10^{-23}\,J\,K^{-1} \cdot 700K}{1.67 \cdot 10^{-27}\,kg}} \approx 1.01\,m\,s^{-1}$

   $u(O) = 8.58\,m \cdot s^{-1}$

   The much lower probable escape velocity of the hydrogen atom means that it is more likely to escape than an oxygen atom.

   From problem text: $E_{pot} = g\,mR$
   $E_{kin} = \frac{1}{2} m\,v^2$

   Assume equillibrium of the energies

   $\frac{1}{2} mv^2 = m\,g\,R$

   where $R \approx 6371\,km = 6371000\,m$ is the Earth radius taken from Lectures in Meterology.

   We obtain after rearranging

   $v_{escape} = \sqrt{2\,g\,R} = \sqrt{2 \cdot 9.81 ms^{-2} 6371000m} \approx 11200 m/s \approx 11.2\,km\,s^{-1}$.

   $v_{escape,Mercury} = \sqrt{2 \cdot 3.8 ms^{-2} 2400000m} \approx 4271 m/s$

   $v_{escape,Mars} = 5028.03\,m/s$

   The graviation must exceed the escaping velocity of the respective gases. Mercury has no atmosphere because its gravitation is too small to hold an atmosphere and its closer vicinity to the Sun most likely means also a higher temperature. Mars can hold a thin atmosphere.

   Mass cancels out, i.e. the escape velocities are independent of the mass of the object, while the probable escape velocities are dependent of the object's mass.

   Note that there are also other possibilities to determine escape velocity. Using the equation for the gravity force $F = \gamma \frac{mM}{R^2}$ where $\gamma = 6.674 \cdot 10^{-11} Nm^2 kg^{-2}$ is the universal gravity constant, $M = 5.97 \cdot 10^{24} kg$ is the Earth's mass, and m is the mass of the object. Derivation of the force with respect to R leads to $E_{gravity} = \gamma \frac{m\,M}{R}$. By using the kinetic energy of the object $E_{kin} = \frac{1}{2} mv^2$ and setting the energies equal provides after some little algebra

   $v = \sqrt{\frac{2\gamma M}{R}} = 11.2 km/s$

i.e. the same result as given above. Analogously the calculation can be carried out for Mercury and Mars, but here the mass of that planets have to be used.Think about how you could use the above knowledge to determine the mass of a planet or moon. Hint: Compare the potential energy and the energy of gravity.

2. Given: $m = 5.14 \cdot 10^{18}\,kg$
   $R = 6.37 \cdot 10^6\,m$
   $g = 9.81\,m\,s^{-2}$

   The definition of pressure is gravity force per unit area. Gravity force is given as

   $F = m\,g$

   By assuming the Earth as a sphere the area is

   $A = 4\,\pi R^2$

   Thus, we have

   $p = \frac{m\,g}{4\,\pi R^2}$

   By plugging in the values we obtain

   $p = \frac{5.14 \cdot 10^{18}\,kg\,9.81\,m\,s^{-2}}{4 \cdot 3.14 \cdot (6.37 \cdot 10^6\,m)^2} \sim 98938\,Pa$

   This value is lower than the means sea-level pressure. The discrepancy is due to the assumption of the Earth geoide as a sphere. The distance to the center is larger at the equator than at the poles due to the Earth's rotation.Consequently, the mean radius under- and overestimates the actual distance at these lacations, respectively. Furthermore, we assumed the acceleration of gravity as a constant. However, it is actually a function of location and height due to the heterogeneous composition of the Earth.

1. Recall the average temperatures at the stratopause in the global mean, at the winter and summer pole? -3^oC, -24^o, 12^oC
2. Explain how the trace gas-, energy-, and water cycle interact.
3. Recall which part of the atmosphere shields the Earth from harmful UV radiation?
4. Explain why is it impossible to define an absolute top of the atmosphere?
5. Describe the vertical structure of the atmosphere by temperature, pressure, and composition.
6. Explain why temperatures in the thermosphere are not comparable with those of the troposhere?
7. Explain the composition of the atmosphere by means of the atmospheric temperature and pressure.
8. Which layer of the Earth's atmosphere contains charged particles, why, and what do they cause?
9. From the color of the aurora can you conclude on the gases of a planet's atmosphere. How should the aurora look like to be a sign for a planet with life?
10. Explain the difference between the escape and probable escape velocity.