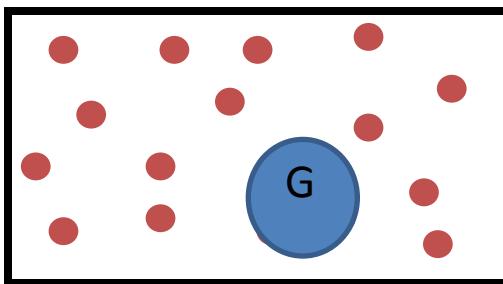
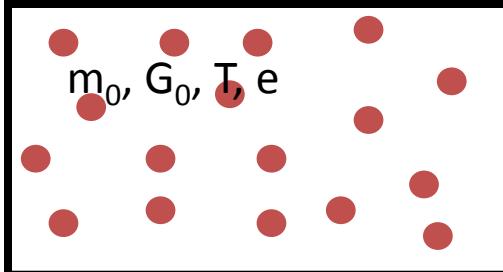


Unit 9

Cloud microphysics

Nicole Mölders

1. Nucleation



Work needed to build surface around droplet

Exchange of energy by water molecules going into the droplet

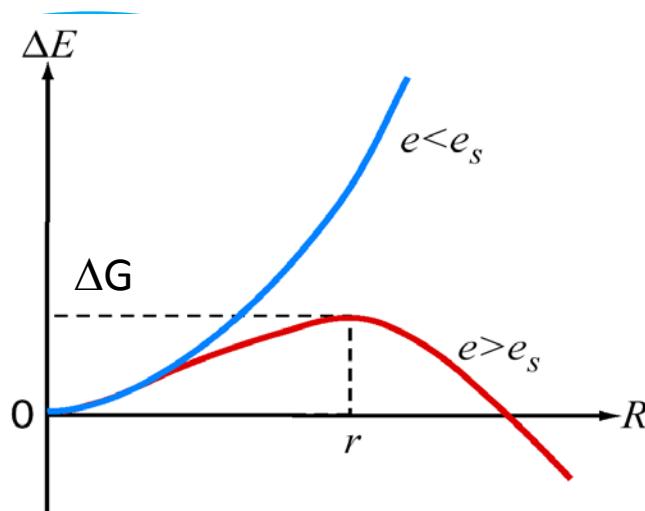
$$G_0 = G_v(e, T) m_0$$

$$\Delta G = G - G_0 = 4\pi r^2 \sigma - \frac{4}{3}\pi r^3 n_w (\mu_v - \mu_w)$$

$$\mu_v - \mu_w = kT \ln\left(\frac{e}{e_s}\right)$$

$$r_c = \frac{2\sigma}{n_w k T \ln \frac{e}{e_s}} = \frac{2\sigma}{\rho_w R_v T} \ln \frac{e}{e_s(T)} \quad \text{Kelvin eq.}$$

$$e = e_s \exp\left(\frac{2\sigma}{n_w k T r_c}\right) = e_s \exp\left(\frac{2\sigma}{\rho_w R_v T r_c}\right).$$



THM: minimization of Gibbs free energy
 $G = G_v(T, e) + G_w(T_w, e) m_w + \sigma A$

THM: Curvature plays a role in droplet formation

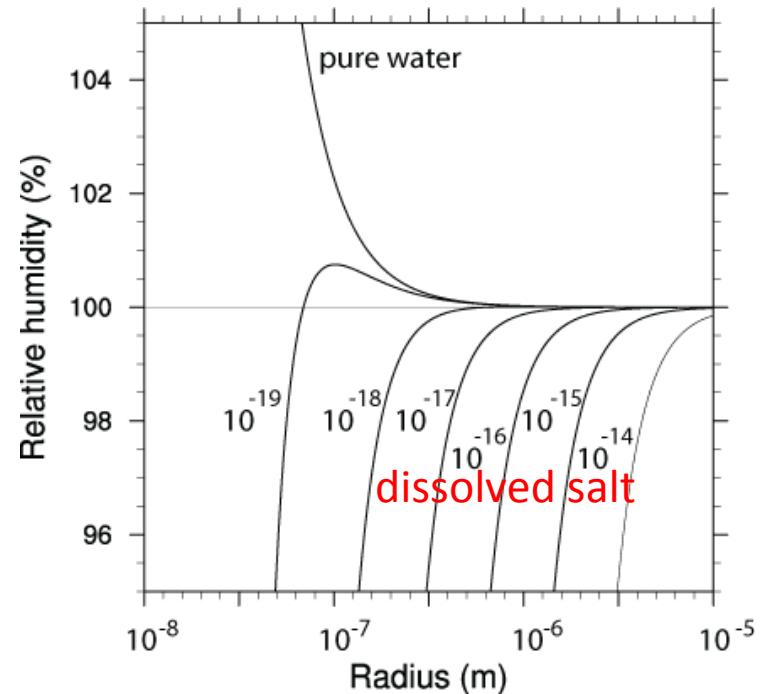
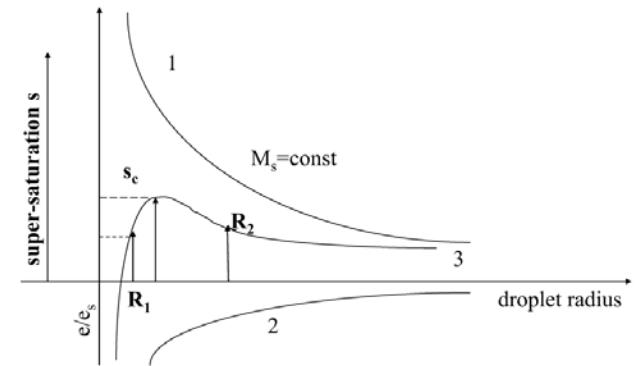
Kelvin effect + Raoult effect = Köhler theory

$$1 - \frac{b}{r^3} \text{ with } b = \frac{3im_s M_w}{4\pi\rho_l M_s}$$

Raoult's law

$$S = \left(1 + \frac{a}{r}\right) \left(1 - \frac{b}{r^3}\right) \approx 1 + \frac{a}{r} - \frac{b}{r^3}$$

curvature term solution term



2. Diffusion

$$\frac{\partial \rho_v}{\partial t} = \nabla \cdot (D_v \nabla \rho_v) = D_v \nabla^2 \rho_v$$

$$\nabla^2 \rho_v(R) = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 \frac{\partial \rho_v}{\partial R}) = 0 .$$

$$\rho_v(R) = \rho_v(\infty) - \frac{r}{R} (\rho_v(\infty) - \rho_v(r)) .$$

$$\frac{dm}{dt} = 4\pi r^2 D_v \frac{d\rho_v}{dR} \Big|_r \quad \frac{dm}{dt} = 4\pi r D_v (\rho_v(\infty) - \rho_v(r))$$

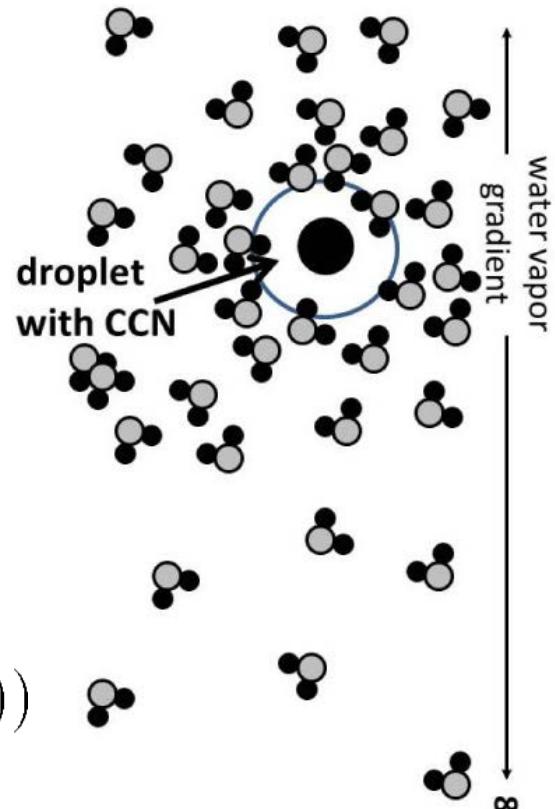
$$L_v \frac{dm}{dt} = 4\pi \kappa_a r (T(r) - T(\infty))$$

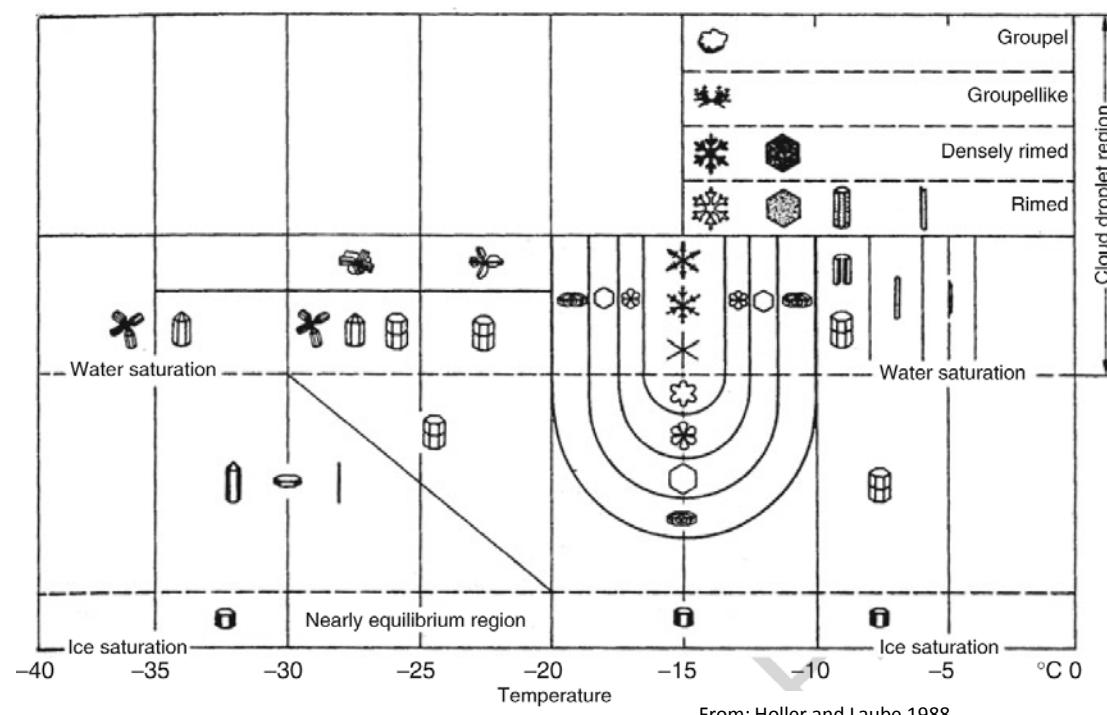
$$\frac{d\rho_{v,s}}{\rho_{v,s}} = \frac{L_v}{R_v} \frac{dT}{T^2} - \frac{dT}{T}$$

$$\frac{dm}{dt} = \frac{4\pi rs}{F_K + F_D}$$

$$\frac{dm}{dt} = \frac{4\pi r}{F_K + F_D} \left(s - \frac{a}{r} + \frac{b}{r^3} \right)$$

.





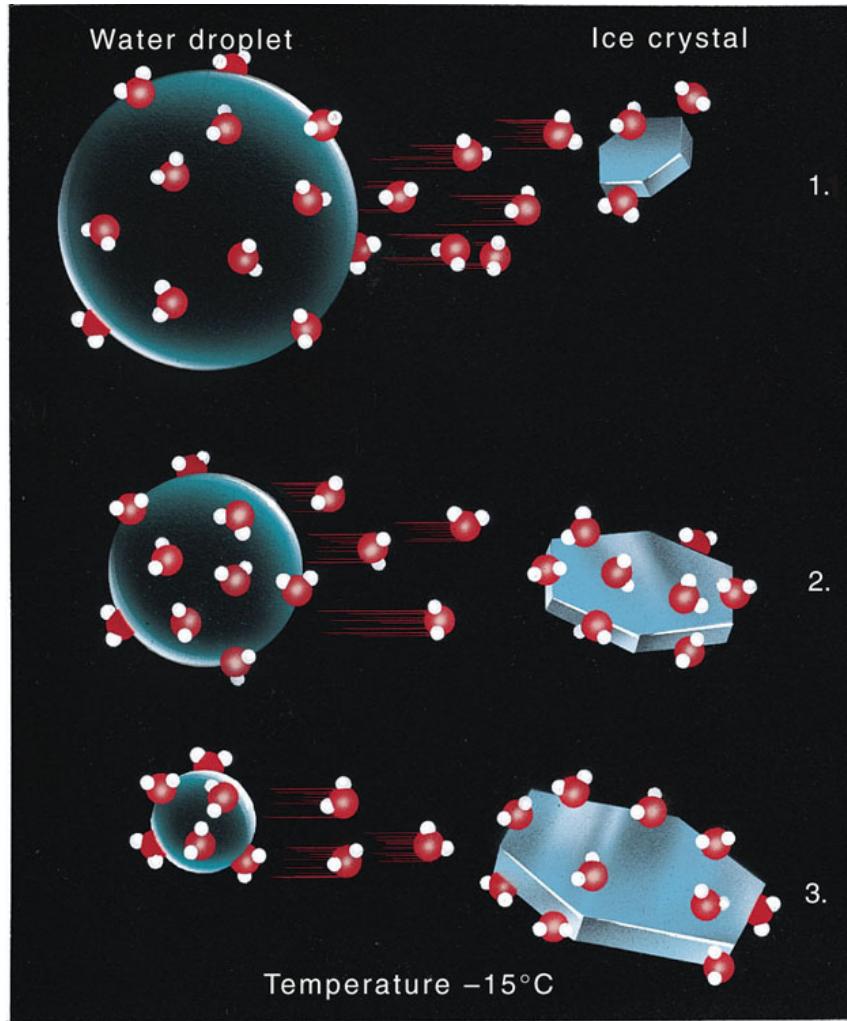
From: Holler and Laube 1988

Snow crystal classification system of Magono-Lee

	N1a Elementary needle		C1f Hollow column		P2b Stellar with sectorlike ends
	N1b Bundle of elementary needles		C1g Solid thick plate		P2c Dendrite with plates at ends
	N1c Elementary sheath		C1h Thick plate of skeletal form		P2d Dendrite with sectorlike ends
	N1d Bundle of elementary sheaths		C1i Scroll		P2e Plate with simple extensions
	N1e Long solid column		C2a Combination of bullets		P2f Plate with sector extensions
	N2a Combination of needles		C2b Combination of columns		P2g Plate with dendrite extensions
	N2b Combination of sheaths		P1a Hexagonal plate		P3a Two branches
	N2c Combination of long solid columns		P1b Sector plate		P3b Three branches
	C1a Pyramid		P1c Broad branch		P3c Four branches
	C1b Cup		P1d Stellar		P4a Broad branch with 12 branches
	C1c Solid bullet		P1e Ordinary dendrite		P4b Dendrite with 12 branches
	C1d Hollow bullet		P1f Fernlike dendrite		P5 Malformed crystal
	C1e Solid column		P2a Stellar with plates at ends		P6a Plate with spatial branches

	P6b Plate with spatial dendrites		CP3d Plate with scrolls at ends		R3c Graupel-like with nonrimmed extensions
	P6c Stellar with spatial plates		S1 Side planes		R4a Hexagonal graupel
	P6d Stellar with spatial dendrites		S2 Scalelike side planes		R4b Lump graupel
	P7a Radiating assemblage of plates		S3 Side planes with bullets and columns		R4c Conelike graupel
	P7b Radiating assemblage of dendrites		R1a Rimed needle		I1 Ice particle
	CP1a Column with plates		R1b Rimed columnar		I2 Rimed particle
	CP1b Column with dendrites		R1c Rimed plate or sector		I3a Broken branch
	CP1c Multiple capped column		R1d Rimed stellar		I3b Rimed broken branch
	CP2a Bullet with plates		R2a Densely rimed plate or sector		I4 Miscellaneous
	CP2b Bullet with dendrites		R2b Densely rimed stellar		G1 Minute column
	CP3a Stellar with needles		R2c Stellar with rimed spatial branches		G2 Germ of skeletal form
	CP3b Stellar with columns		R3a Graupel-like snow of hexagonal type		G3 Minute hexagonal plate
	CP3c Stellar with scrolls at ends		R3b Graupel-like snow of lump type		G4 Minute stellar
					G5 Minute assemblage of plates
					G6 Irregular germ

Bergeron-Findeisen process



3. Sedimentation

$$\rho \cdot \frac{4}{3}\pi \cdot r^3 \cdot g = \pi \cdot r^2 \cdot v \cdot \alpha$$

$$v = \frac{4}{3} \rho \cdot g \cdot r = v(r)$$

•TABLE 7.1

Terminal Velocity of Different-Size Particles Involved in Condensation and Precipitation Processes

TERMINAL VELOCITY			
Diameter (μm)	m/sec	ft/sec	Type of Particle
0.2	0.0000001	0.0000003	Condensation nuclei
20	0.01	0.03	Typical cloud droplet
100	0.27	0.9	Large cloud droplet
200	0.70	2.3	Large cloud droplet or drizzle
1000	4.0	13.1	Small raindrop
2000	6.5	21.4	Typical raindrop
5000	9.0	29.5	Large raindrop

Terminal velocity of ice-crystals depends on shape

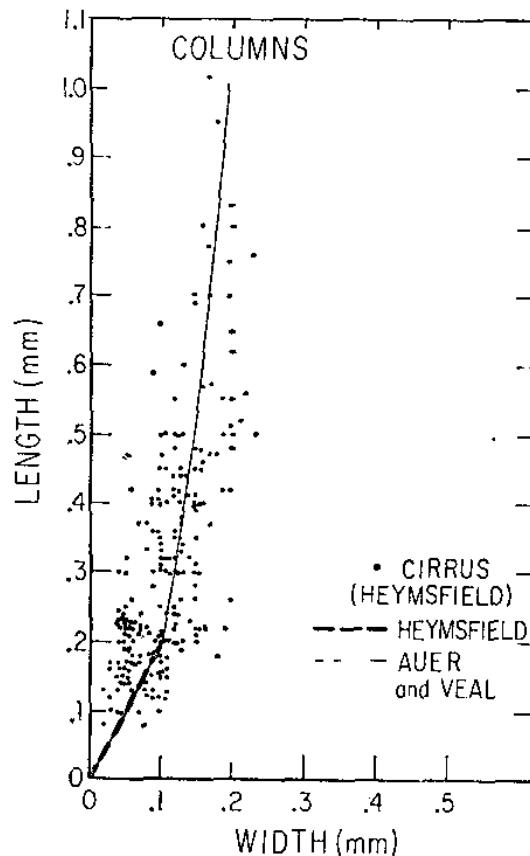


FIG. 2. Column crystal length vs width from crystals samples in cirrus clouds.

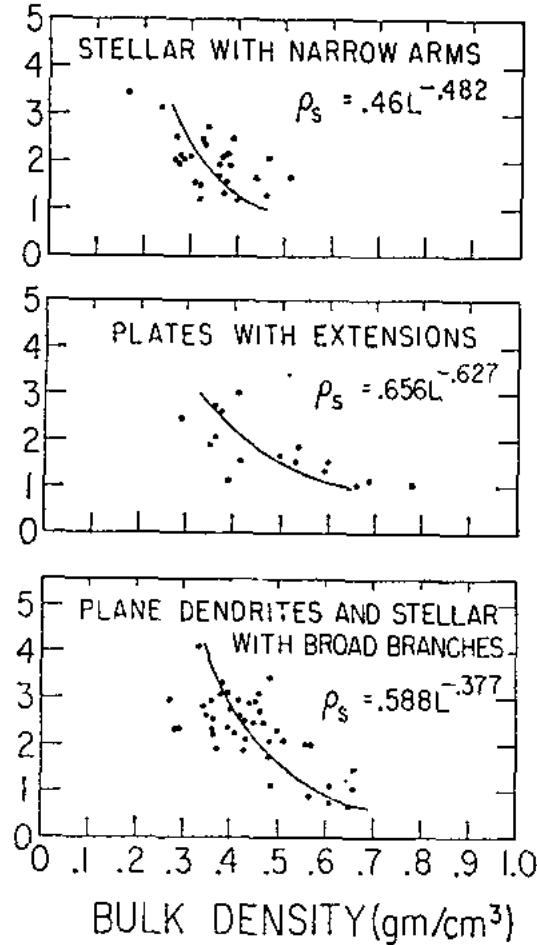


FIG. 5. Dendritic diameter vs density.

From: Heymsfield 1972 (JAS)

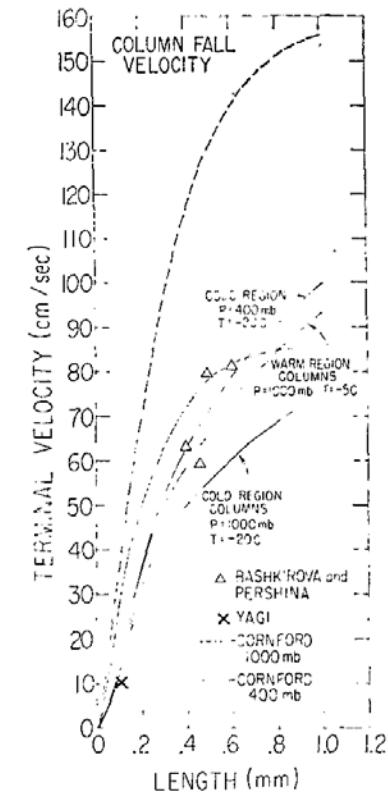


FIG. 6b. Column terminal velocity vs length at 1000 mb and 400 mb.

4. Collection

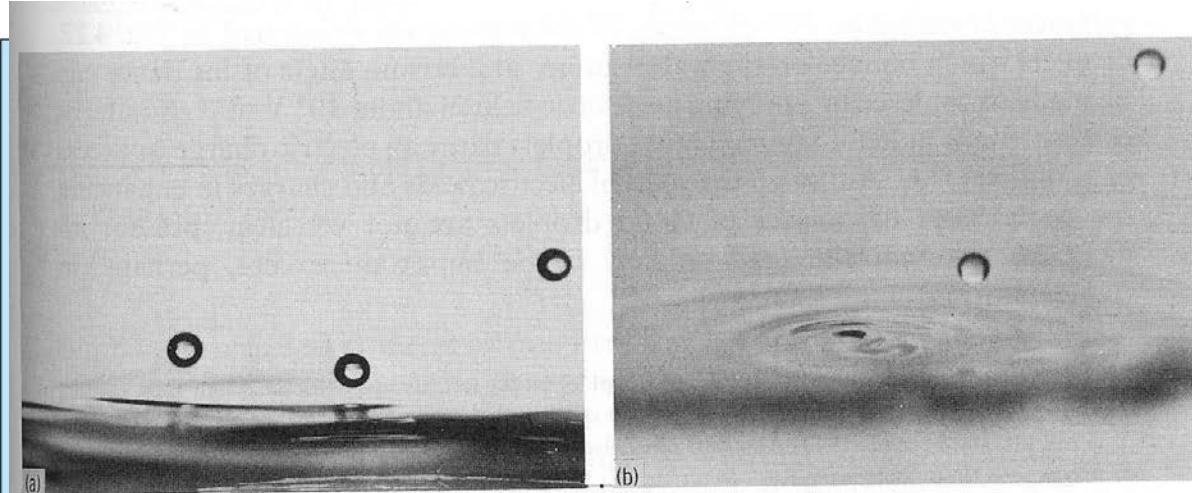
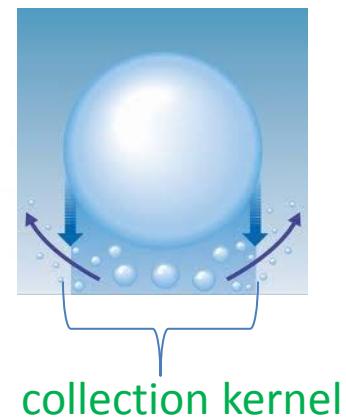
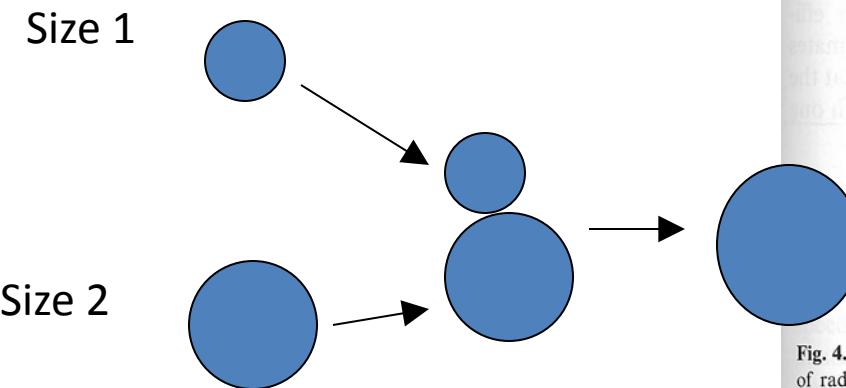


Fig. 4.22 (a) A stream of water droplets (entering from the right), about $100 \mu\text{m}$ in diameter, rebounding from a layer of water. (b) When the angle between the stream of droplets and the surface of the water is increased beyond a critical value, the droplets coalesce with the water. (Photo: P. V. Hobbs.)



collection kernel

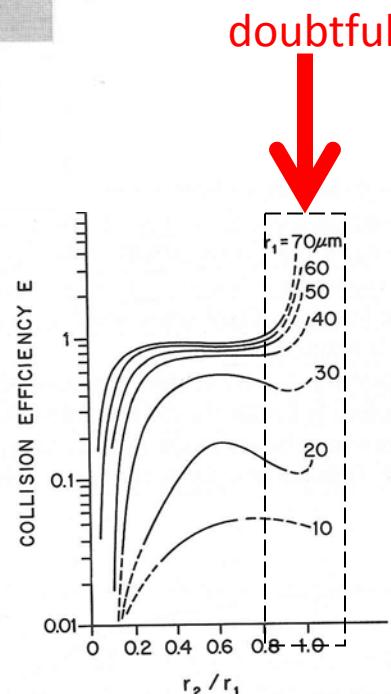


Fig. 4.21 Calculated values of the collision efficiency for collector drops of radius r_1 with droplets of radius r_2 . The dashed portions of the curve represent regions of doubtful accuracy. [From *J. Atmos. Sci.* **30**, 112 (1973).]

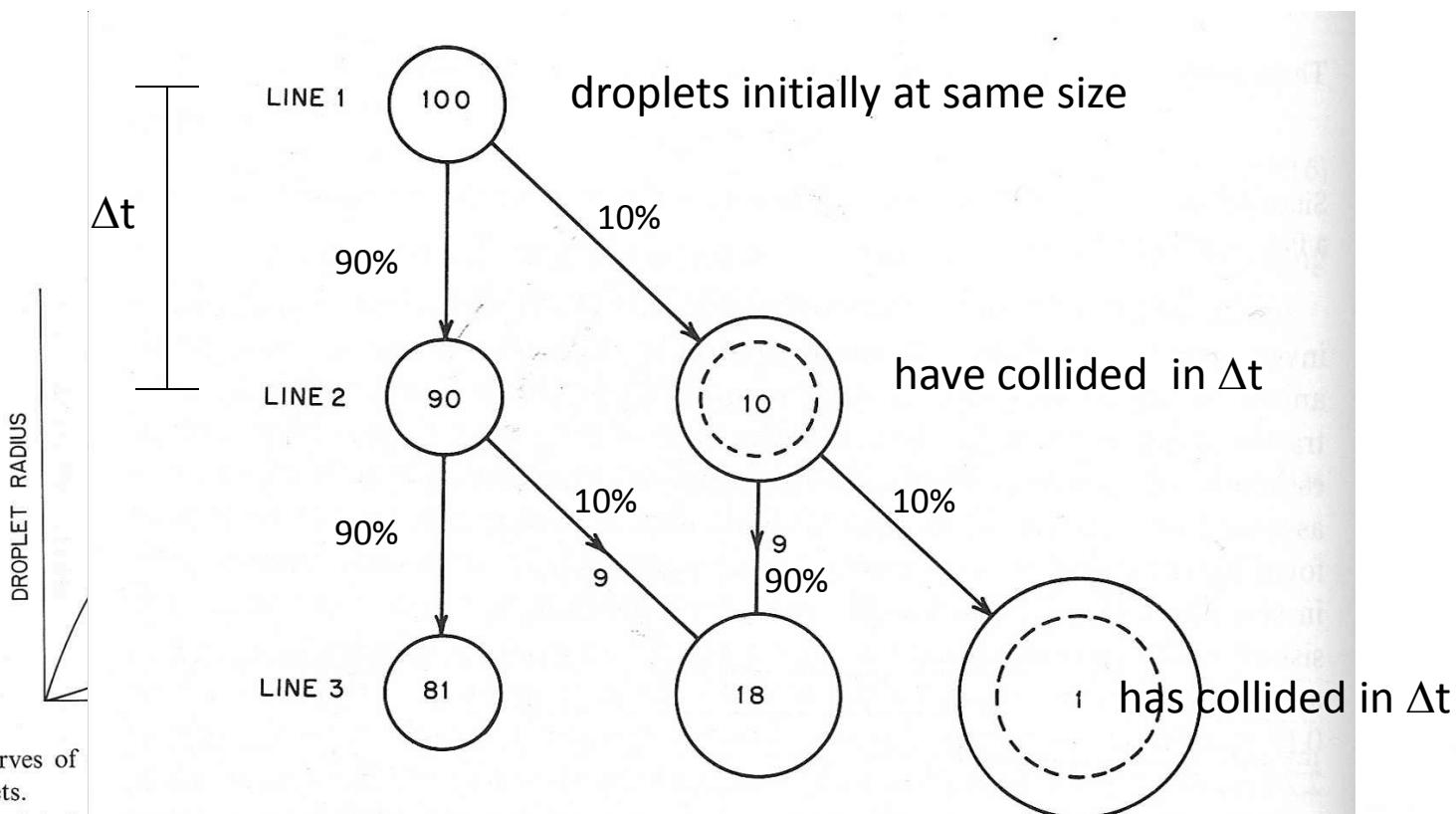


Fig. 4.16 Schematic curves of
(b) coalescence of droplets.

Fig. 4.25 Schematic diagram to illustrate broadening of droplet sizes by statistical collisions.
[From *J. Atmos. Sci.* **24**, 689 (1967).]

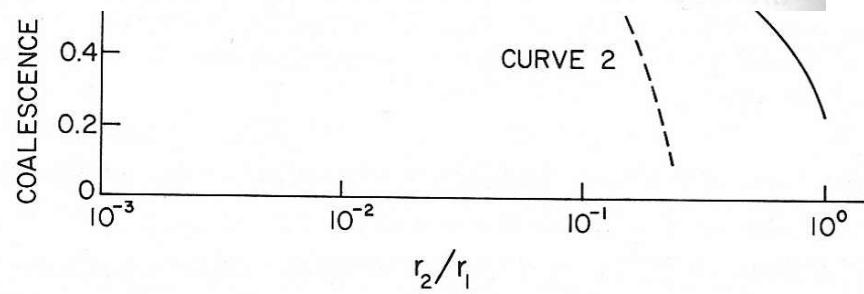
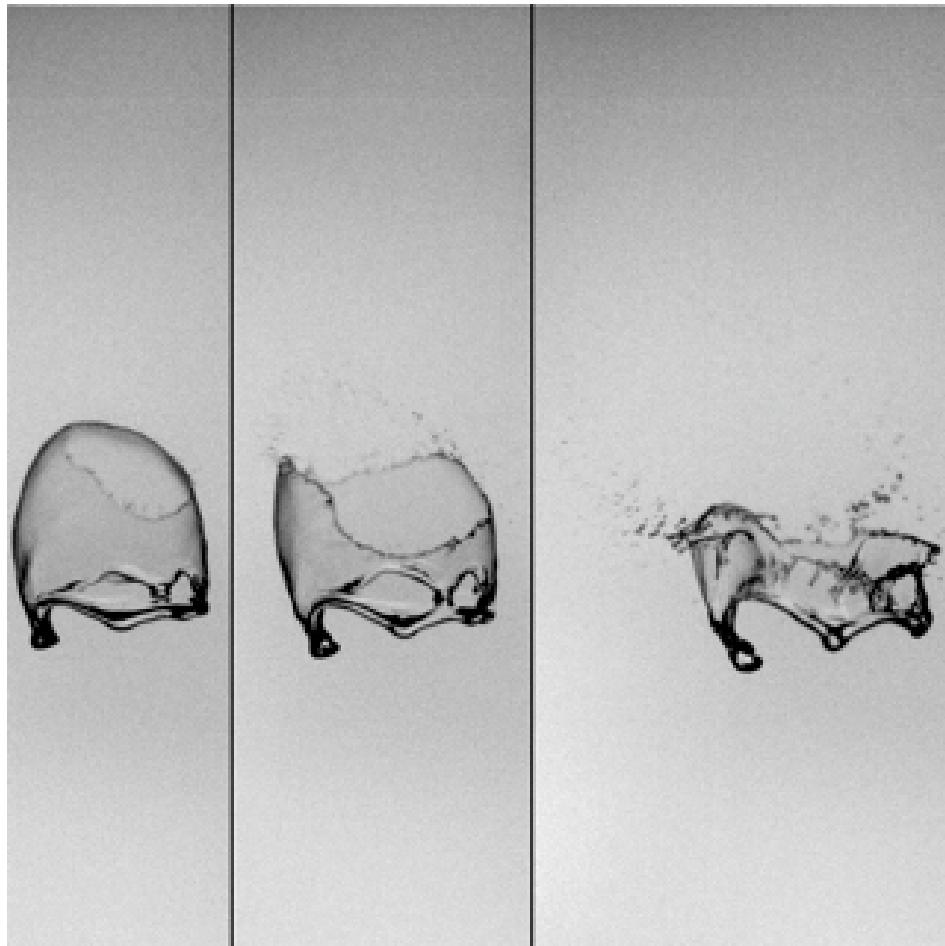


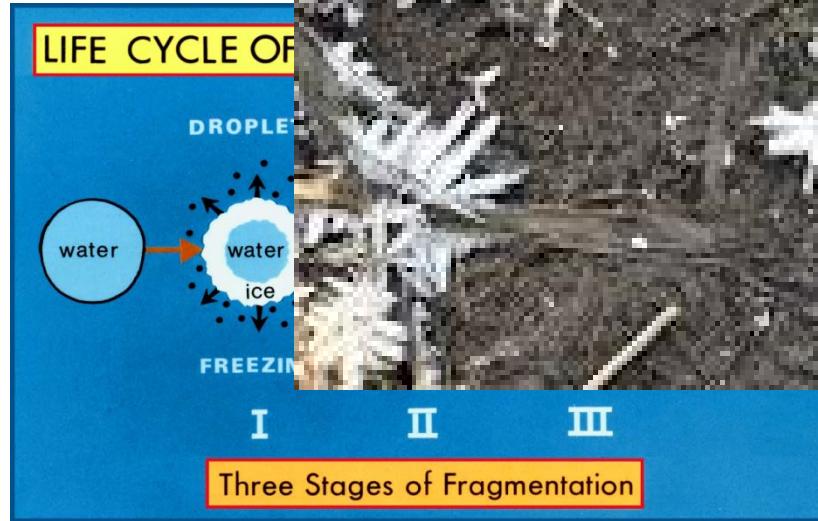
Fig. 4.23 Coalescence efficiencies for a droplet of radius r_2 with a drop of radius r_1 . (—) $400 \mu\text{m} < r_1 < 2000 \mu\text{m}$, $20 \mu\text{m} < r_2 < 100 \mu\text{m}$. [From *J. Geophys. Res.* **76**, 2836 (1971).] (---) r_1 averages $50\text{--}100 \mu\text{m}$. [From *J. Atmos. Sci.* **30**, 944 (1973).]

5. Breakup



From: Scientific American

6. Ice enhancement



7. Melting

$$-L_f \frac{dm}{dt} = 4\pi r \kappa_a (T(\infty) - T_0) v_{fc} + \frac{dm_{col}}{dt} c_w (T_w - T_0) + \frac{dQ_d}{dt}$$



Diffusion of heat towards particle

Change by collection



Gain/loss of heat by
water vapor diffusion



comet

What a cloud model has to consider

Plus mass conservation

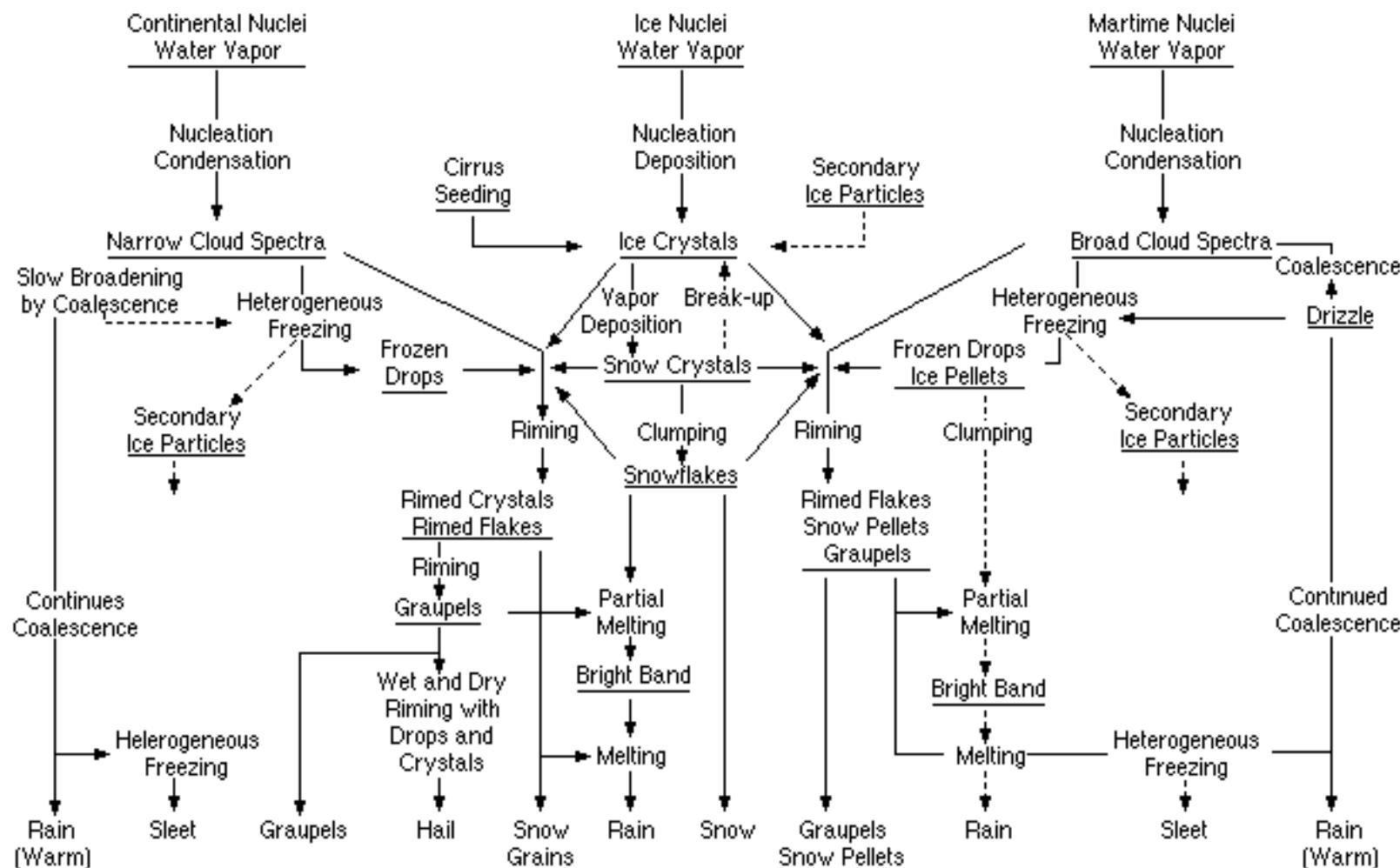


Fig. 1