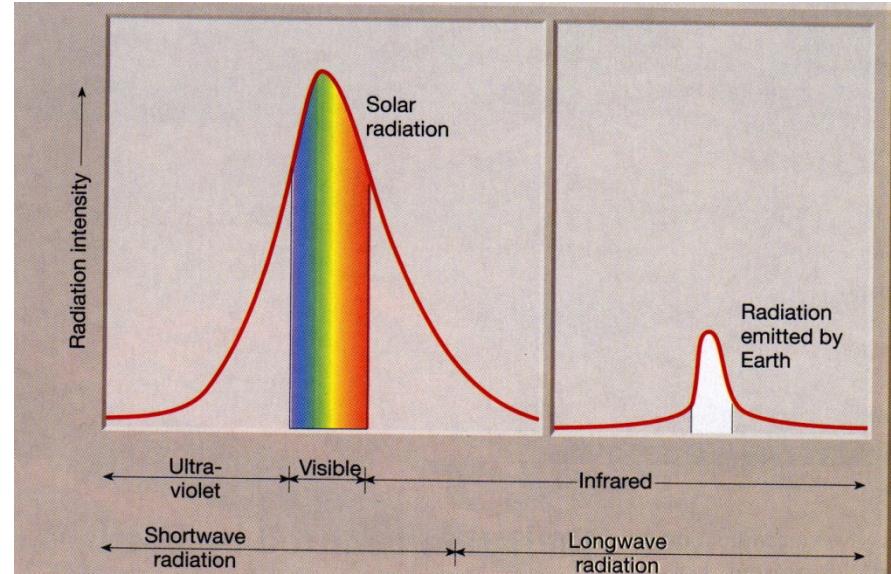


Unit 11

Blackbody radiation

Nicole Mölders

Electromagnetic spectrum



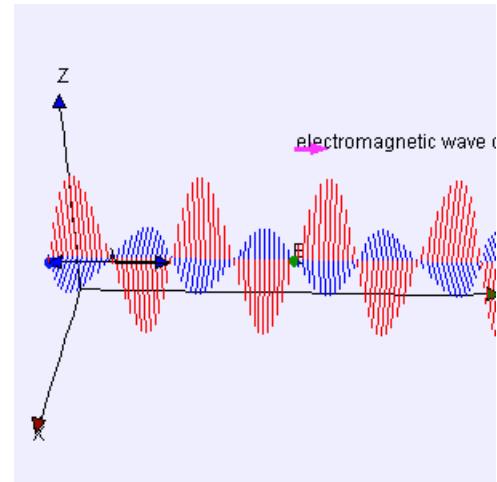
Electromagnetic radiation

$$v=c/\lambda$$

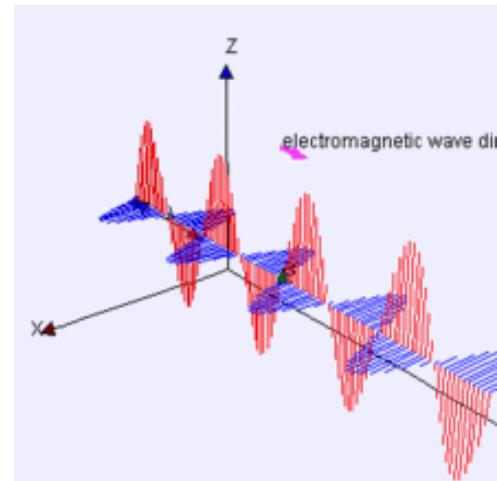
where $c=2.9979 \times 10^8 \text{ m/s}$ is the speed of light in vacuum

$$k=1/\lambda \quad \text{wave number}$$

Note that this wave number differs from that customarily used in physics by the factor of 2π !

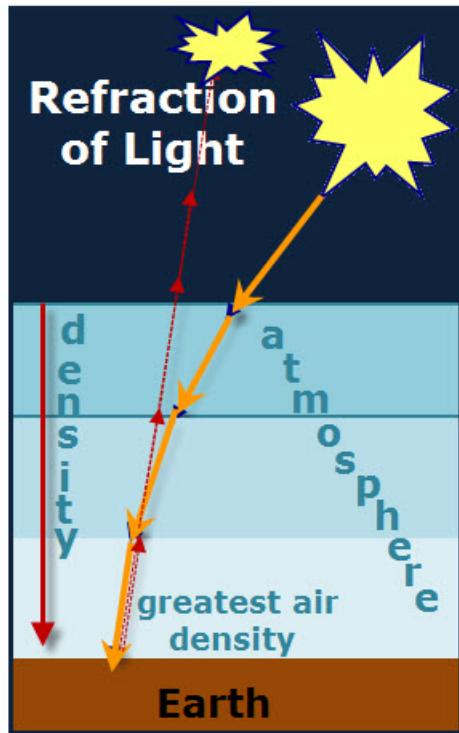
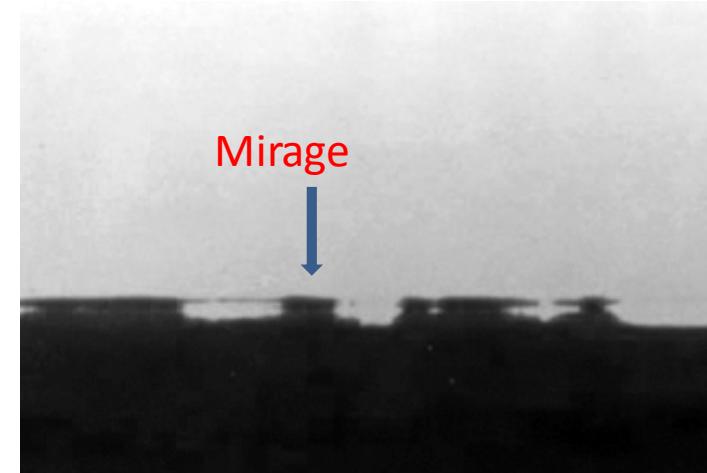


<http://upload.wikimedia.org/wikipedia/commons/a/ad/Electromagneticwave3Dfromside.gif>

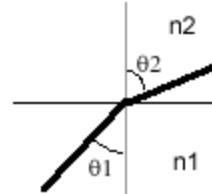


<http://upload.wikimedia.org/wikipedia/commons/thumb/4/4c/Electromagneticwave3D.gif/220px-Electromagneticwave3D.gif>

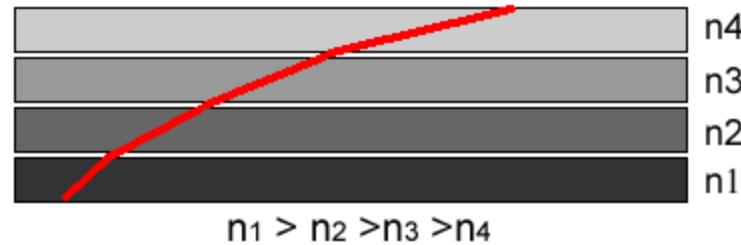
Refraction



Snell's Law



$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

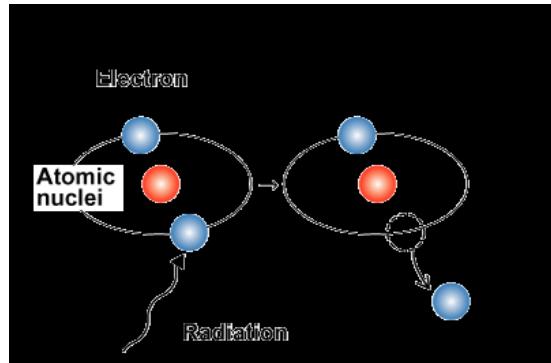


<http://www.mike-willis.com/Images/refraction.gif>

Nomenclature



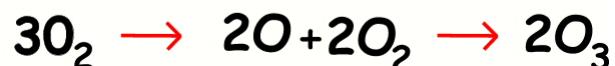
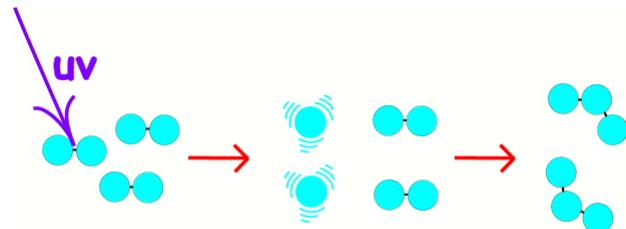
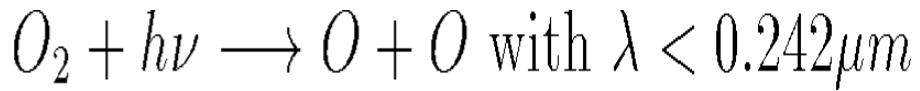
Radiant Flux
Total Power (Watts)



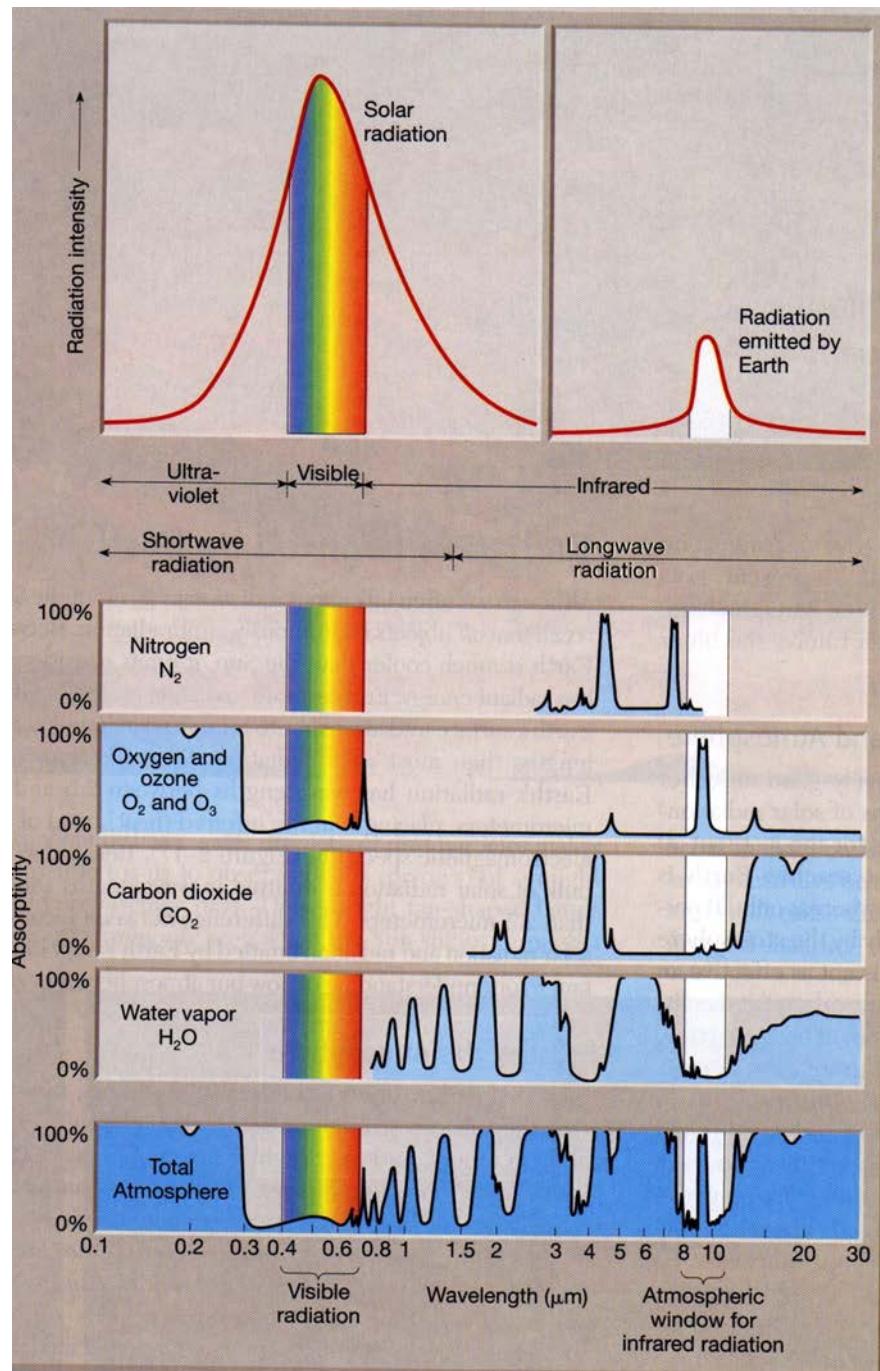
https://www.remnet.jp/english/lecture/b03_01/images/denri.gif

<http://sensing.konicaminolta.asia/wp-content/uploads/2011/06/lc-light-radiant-flux.jpg>

$h=6.6261 \times 10^{-34} \text{ Js}$ is the Planck constant

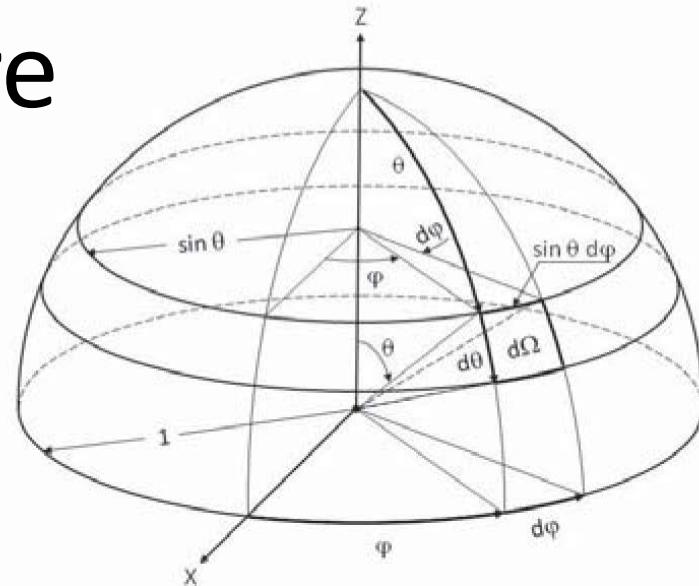


http://resources.yesican-science.ca/trek/scisat/final/images/trans_ozone_make1.jpg



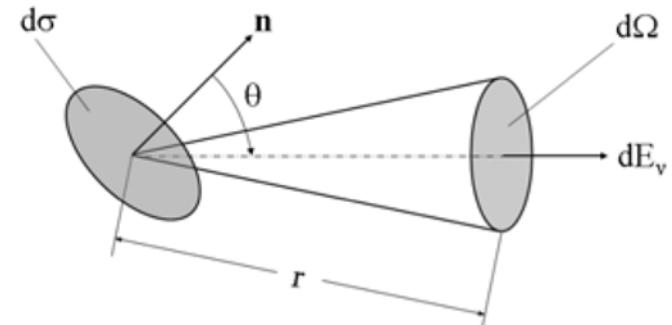
More nomenclature

Solid angle $\Omega = \frac{\sigma}{r^2} = 4\pi \text{ sr}$



$$d\Omega = \frac{d\sigma}{r^2} = \frac{r \, d\theta \, r \sin \theta \, d\phi}{r^2} = \sin \theta \, d\theta \, d\phi = -d\mu \, d\phi$$

where $\mu = \cos \theta$.



$$dE_\nu = I_\nu \cos \theta \, d\sigma \, d\nu \, d\Omega \, dt$$

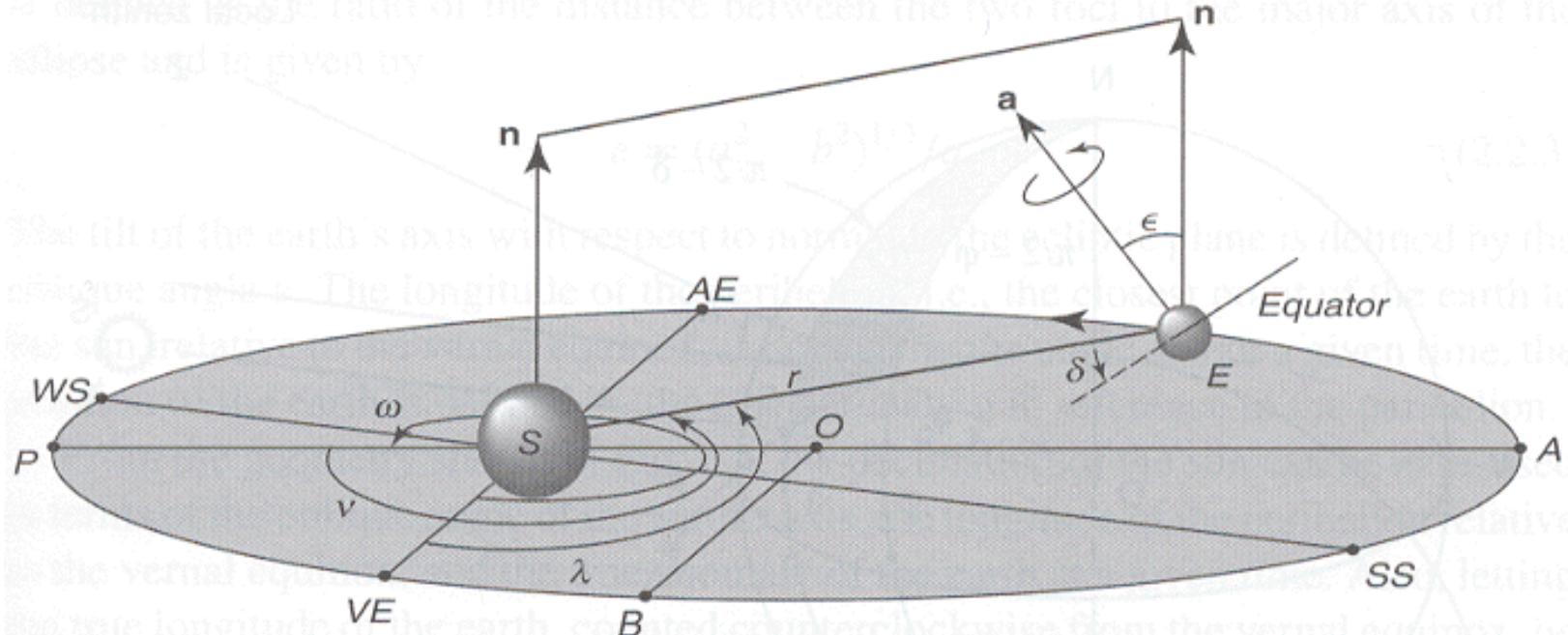
$$I_\nu = \frac{dE_\nu}{\cos \theta \, d\sigma \, d\nu \, d\Omega \, dt}$$

$$I_\lambda = - \frac{c}{\lambda^2} I_\nu$$

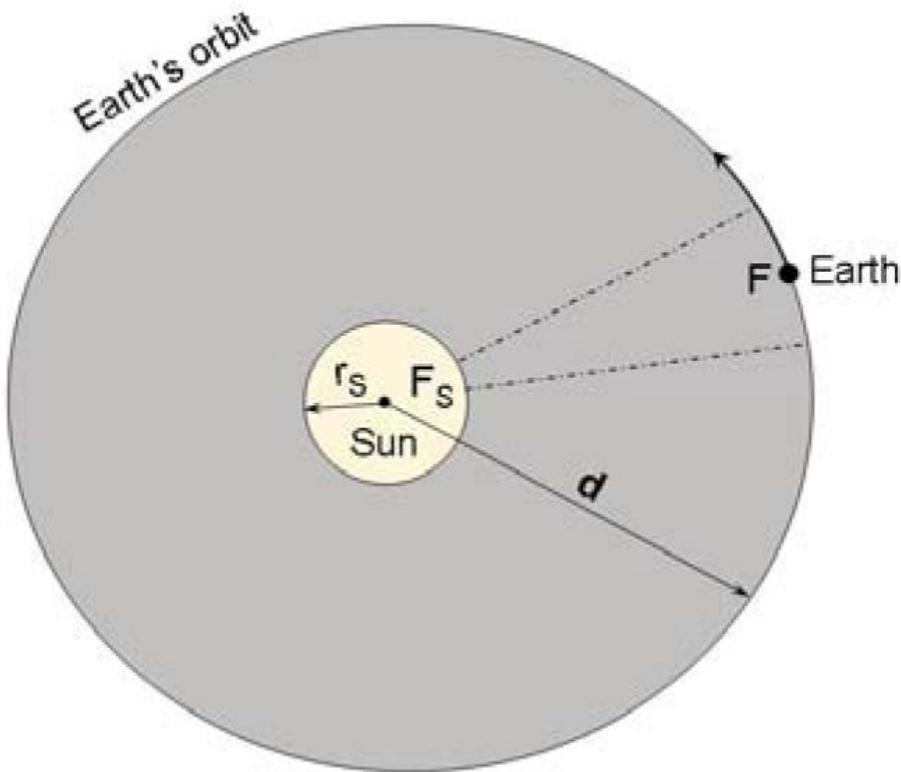
Symbols and units of various radiometric quantities (adopted from Liou, 2002)

Symbol	Quantity	Units
E	Energy	J = Ws
I_λ	Monochromatic intensity	$\frac{J}{m^3 s sr} = \frac{W}{m^3 sr}$
F_λ	Monochromatic flux density	$\frac{J}{m^3 s} = \frac{W}{m^3}$
F	Total flux density	$\frac{J}{m^2 s} = \frac{W}{m^2}$
Φ	Total flux	$\frac{J}{s} = W$

The Sun – Earth Geometry



E Earth, S sun, P *Perihelion*, A *Aphelion*, AE *Autumnal Equinox*, VE *Vernal Equinox*, WS *Winter Solstice*, SS *Summer Solstice*, \mathbf{n} normal vector (perpendicular to *ecliptic plane*), \mathbf{a} is parallel to the Earth's axis, δ *sun declination*, ϵ *oblique angle* of the Earth's axis, ω longitude of the perihelion relative to VE, v true anomaly of the earth at a given time, λ true longitude of the Earth, O center of the ellipse, OA (OP) semi-major axis, OB the semiminor axis, r the distance earth-sun (adopted from Liou, 2002; Kramm 2005).



Solar Irradiance at the Outer Edge of the Atmosphere

$$F = \left(\frac{r_s}{d}\right)^2 F_s$$

$$F_s = \sigma T_s^4$$

$$F = \left(\frac{r_s}{d}\right)^2 \sigma T_s^4$$

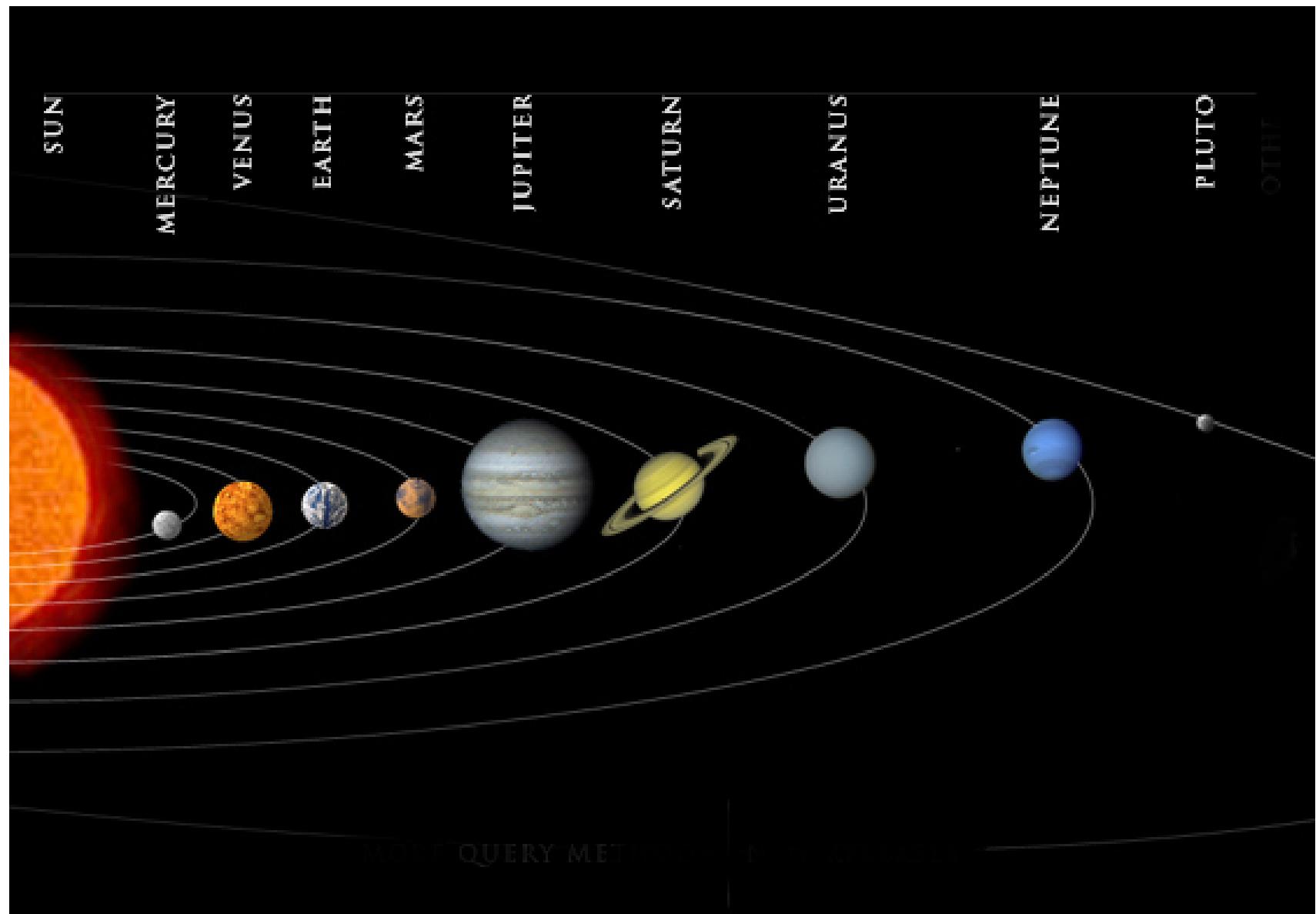
$r_s = 6.96 \times 10^8$ m visible Sun radius
 d actual distance Earth-Sun (ranges from 1.471×10^{11} m at the Perihelion to 1.521×10^{11} m at the Aphelion)
 $F_s = 6.288 \times 10^7$ Wm $^{-2}$ solar exitance
 $\sigma = 5.67 \times 10^{-8}$

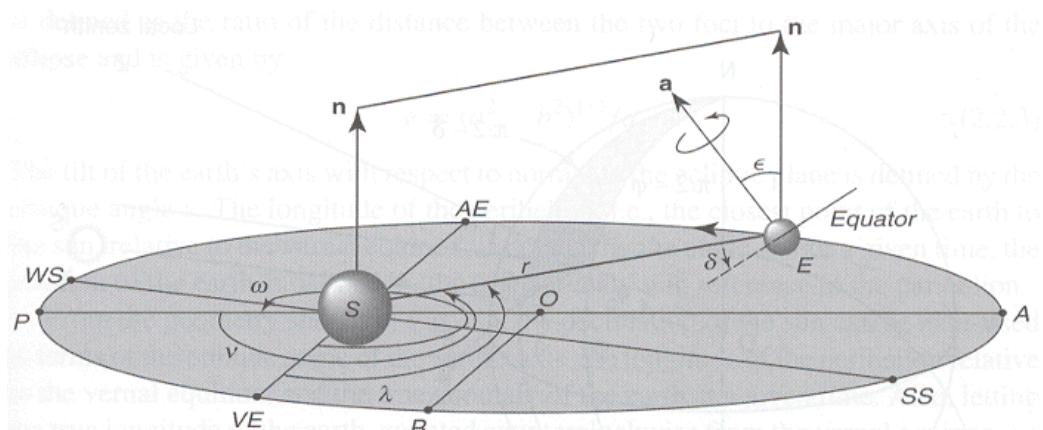
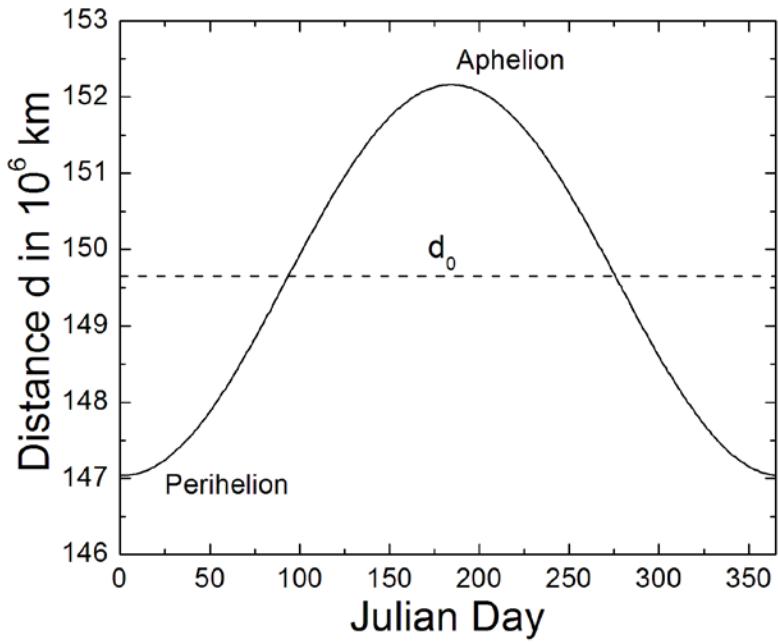
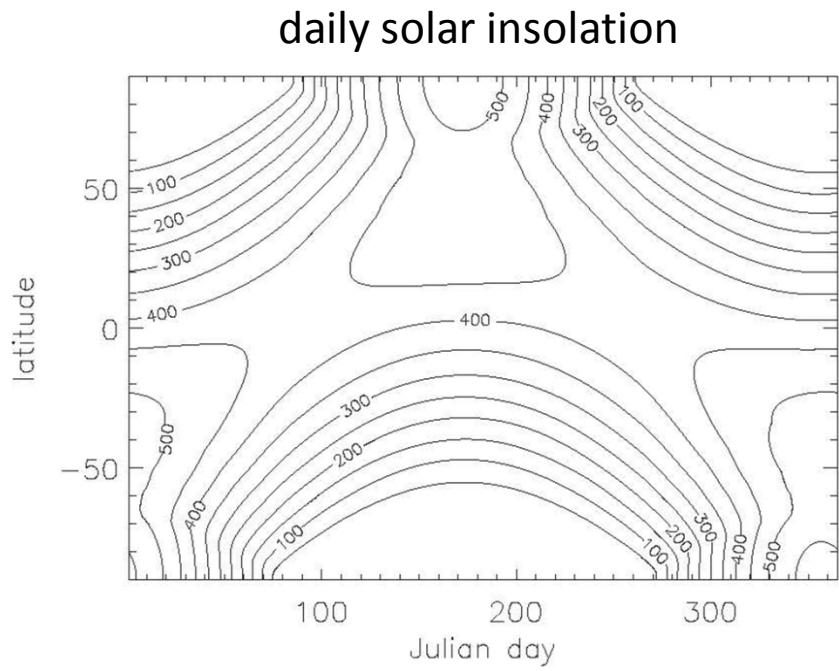
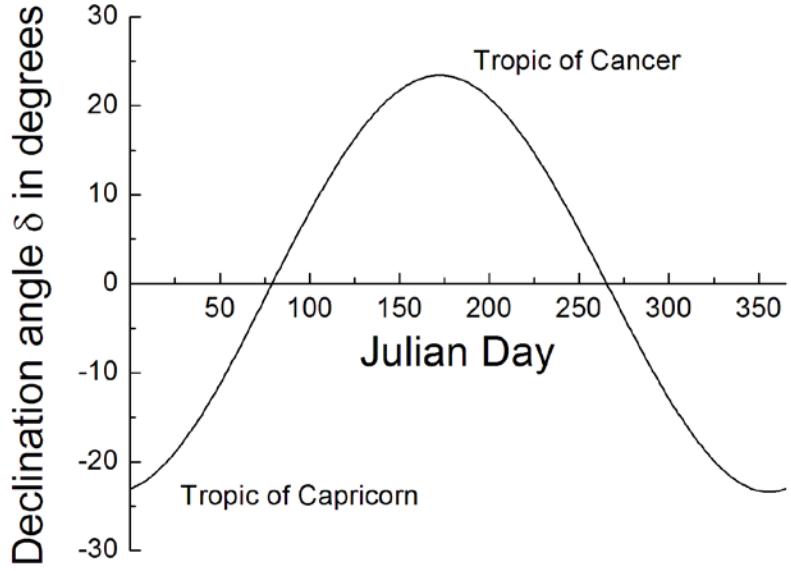
Total radiation flux = const. $4\pi d^2 F = 4\pi r_s^2 F_s$

$$S = \left(\frac{d}{d_0}\right)^2 F$$

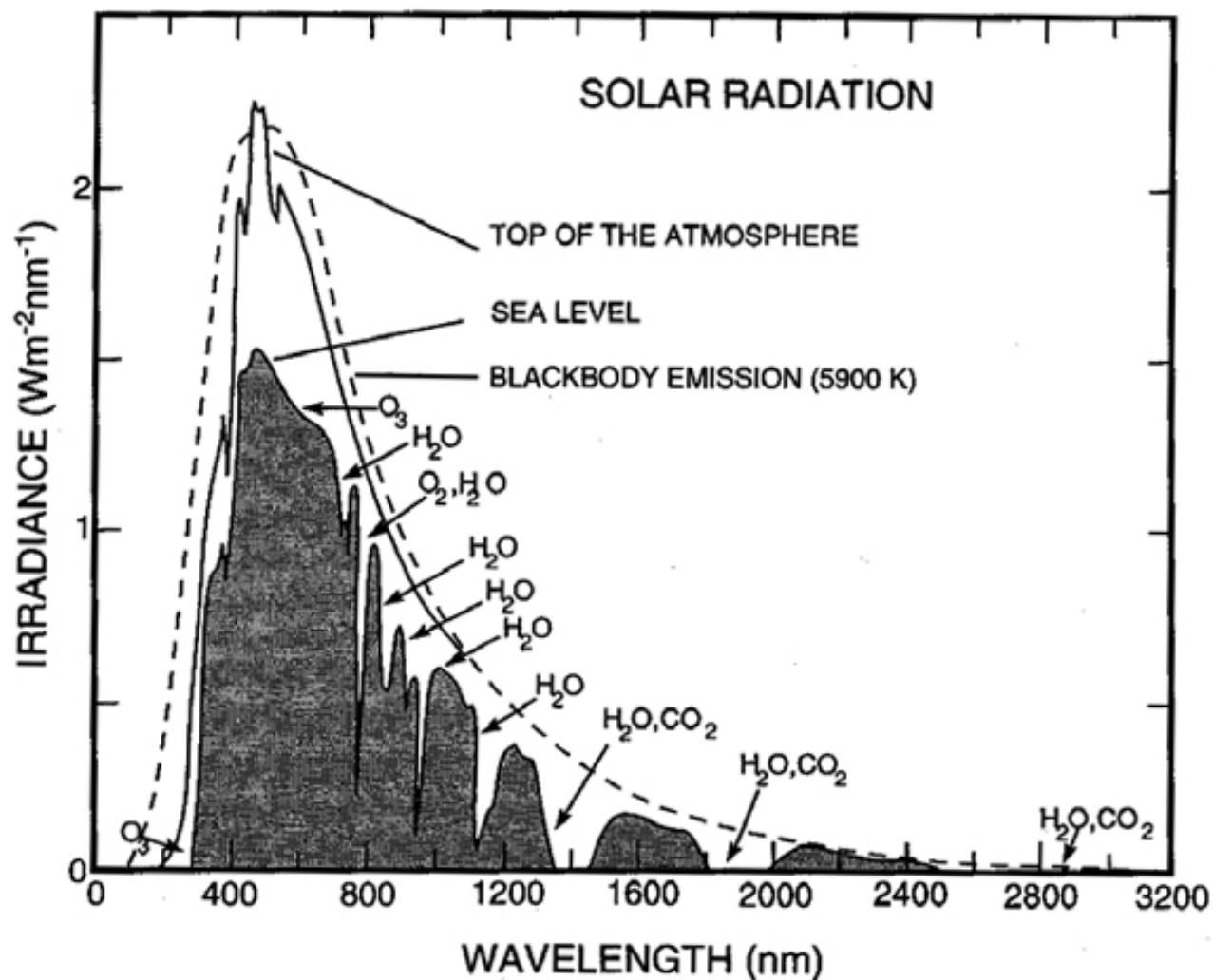
Solar constant $S = 1361 \text{ Wm}^{-2}$ for mean distance of 1.496×10^{11} m = 1 Astronomic Unit

Each planet has its own solar constant...





Solar irradiance at the top of the atmosphere



Kirchhoff's law (1860)

Prerequisite: Thermal equilibrium

$$\frac{E_v}{a_v} = J(v, T)$$

$$J(v, T) = B(v, T) \quad \text{Planck (1900/01)}$$

E_v emissivity

a_v absorptivity

$J(v, T)$ emissive power of a black body ($a_v = 1$)

$$\varepsilon_v = \frac{E_v}{J(v, T)} = a_v$$

relative emissivity

Stefan Boltzmann Law

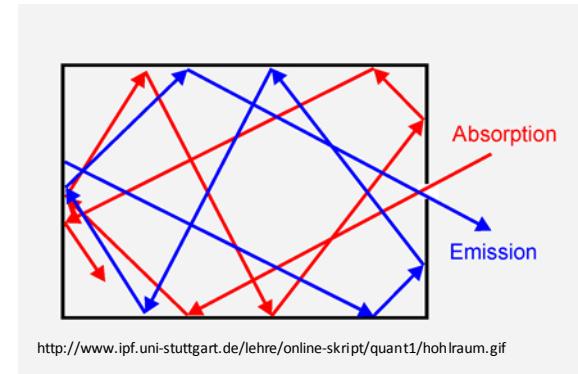
$$F = \sigma T^4$$

blackbody

with F flux of energy (W/m^2)

T temperature (K)

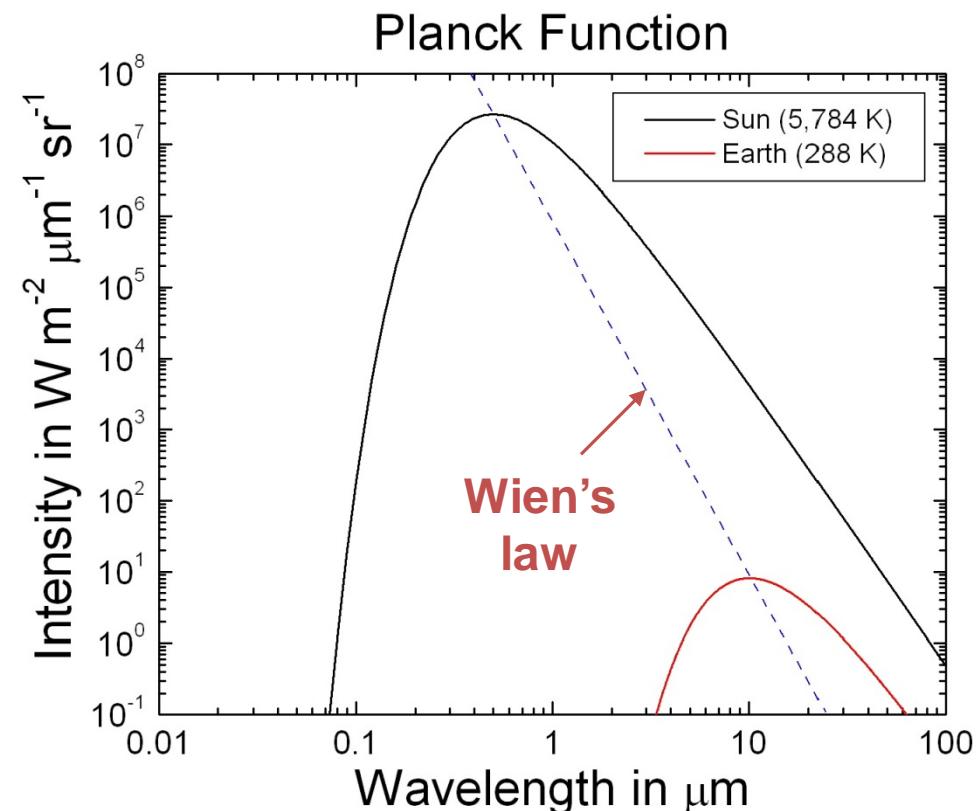
$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ Stefan-Boltzmann constant



$$F = \epsilon \sigma T^4$$

graybody

Planck's Blackbody Radiation Law



$$B_{\lambda}(T) = -\frac{2 h c^2}{\lambda^5 \left\{ \exp\left(\frac{h c}{\lambda k T}\right) - 1 \right\}}$$

$B_{\lambda}(T)$ = monochr. intensity

T = absolute temperature

h = Planck's constant

c = velocity of light in vacuum

k = Boltzmann's constant

λ = wavelength

$$\lambda_{\max} = \frac{2,897 \mu\text{m}}{T}$$

Wien's law