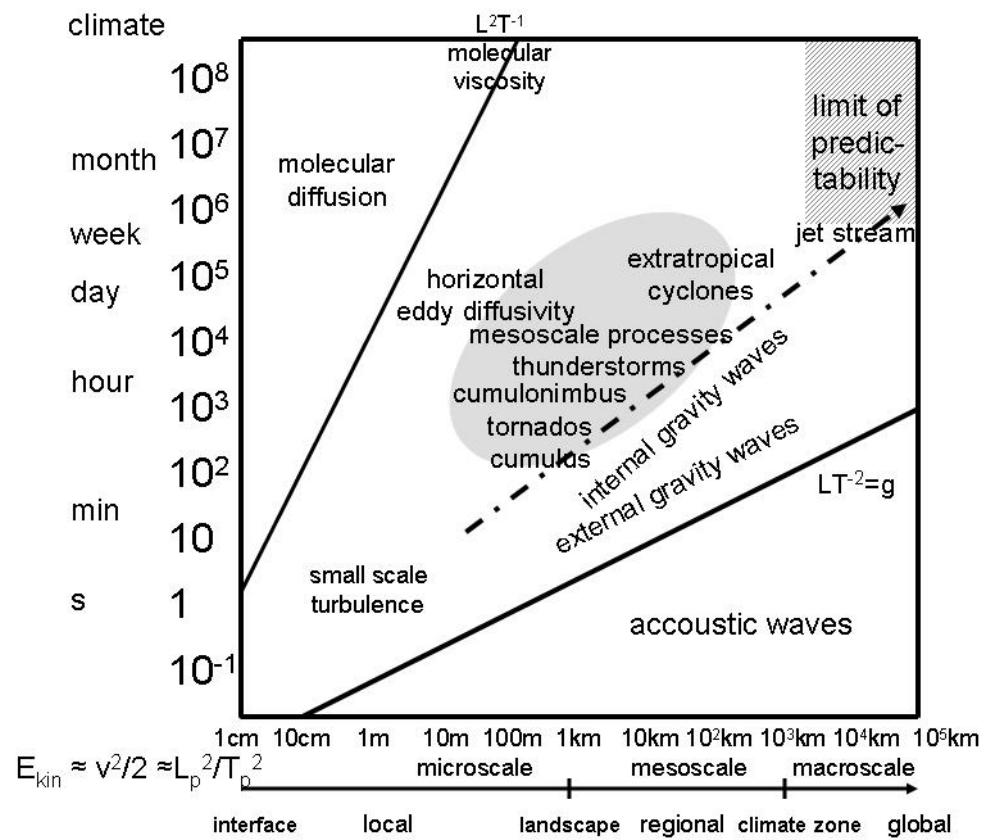


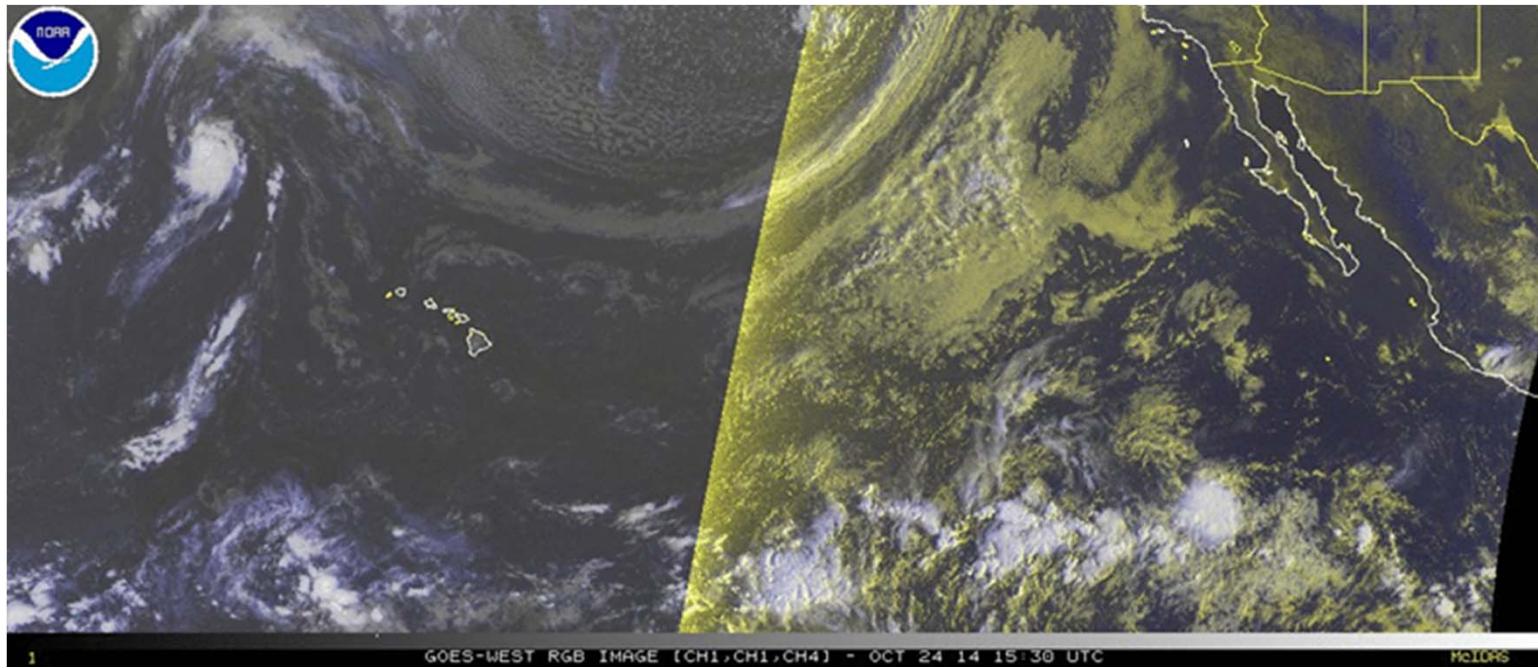
Unit 18

Vorticity, Navier-Stokes and Euler equation
Nicole Mölders

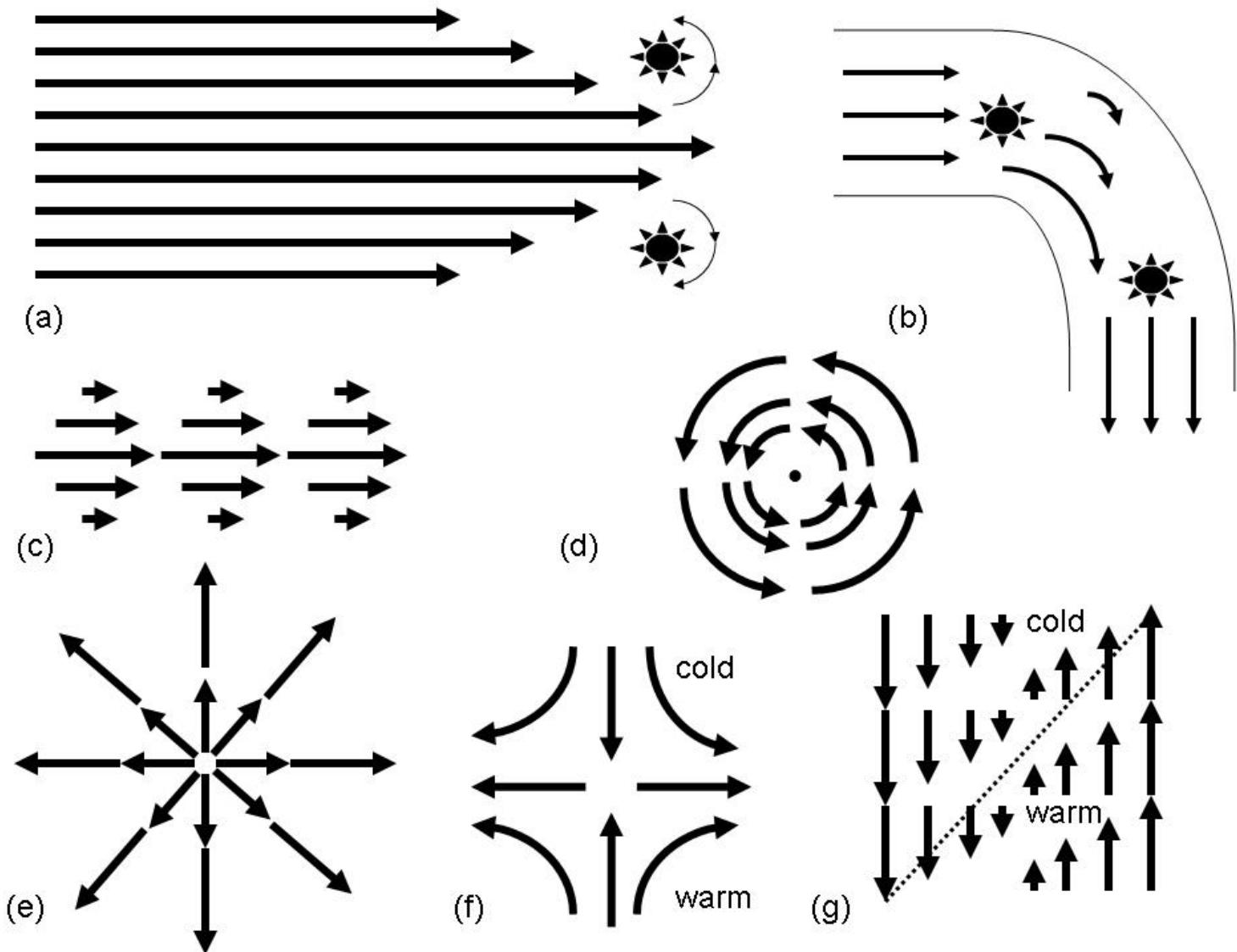
Dynamic meteorology



Kinematics of large-scale flow



Kinematics of large-scale flow

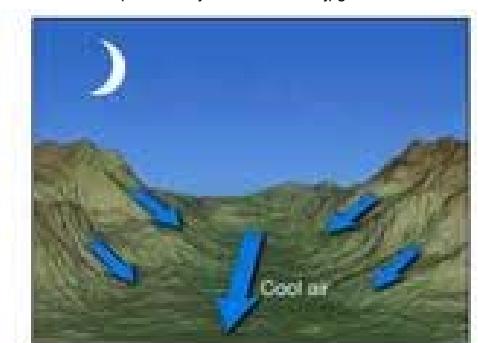
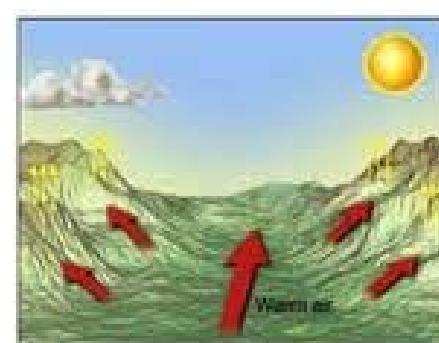
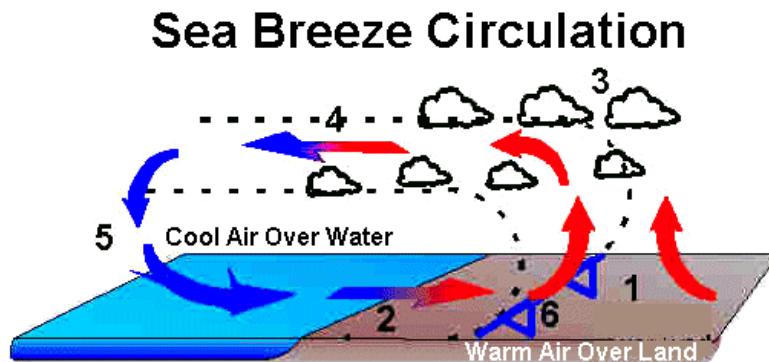
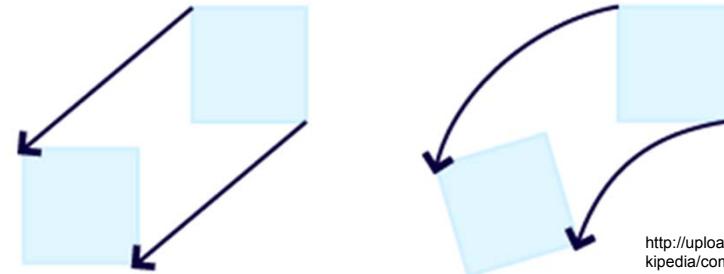


Vorticity in the atmosphere

$$\bar{\omega} = \frac{C}{A}$$

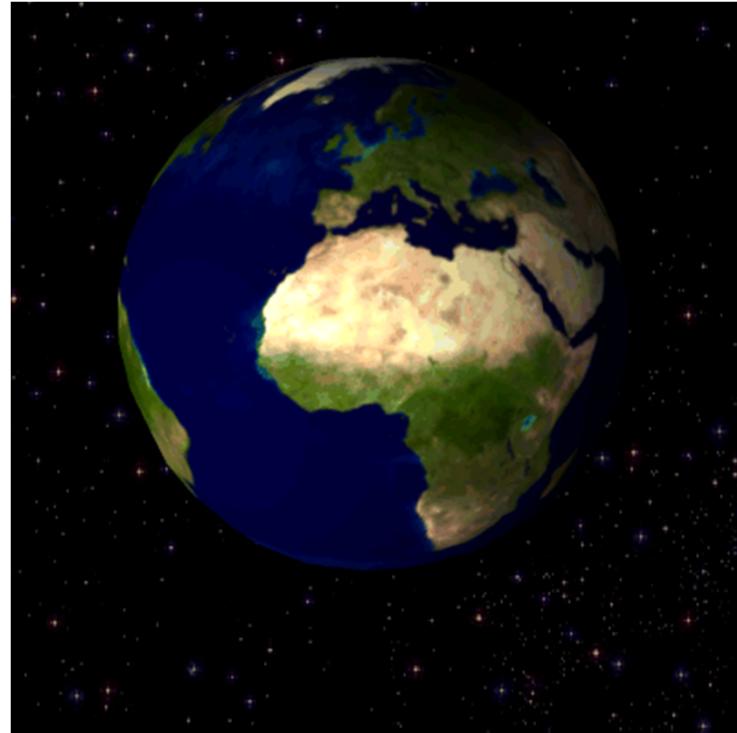
$$\omega = \frac{dC}{dA}$$

$$\vec{\omega} = \nabla \times \vec{v}$$



http://t2.gstatic.com/images?q=tbn:ANd9GcRUUXKPlm3fS_FaC3bb_dvCVlpIDFwAtn3Ts_x--25VixBy6K_t1WI9YtEr

Relative and absolute vorticity



[http://upload.wikimedia.org/wikipedia/commons/2/2c/
Rotating_earth_\(large\).gif](http://upload.wikimedia.org/wikipedia/commons/2/2c/Rotating_earth_(large).gif)

CREDIT: PHL @ UPR Arecibo and the NERC Satellite Receiving Station, Dundee University, Scotland. All images are copyright of EUMETSAT.

Vorticity

Absolute vorticity

$$\nabla \times \mathbf{v}_a = \nabla \times \mathbf{v} + \nabla \times (\boldsymbol{\Omega} \times \mathbf{r}) = \overbrace{\nabla \times \mathbf{v}}^{\text{Relative vorticity}} + \underbrace{2\boldsymbol{\Omega}}_{\text{vorticity of the earth}}$$

$\Omega = 0.7272 \times 10^{-4} \text{ s}^{-1}$

$$\mathbf{k} \cdot (\nabla \times \mathbf{v}_a) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \xi + f = \xi + 2\Omega \sin \phi = \xi_a$$
$$f = 2\Omega \sin \phi$$

Path Without Coriolis Effect



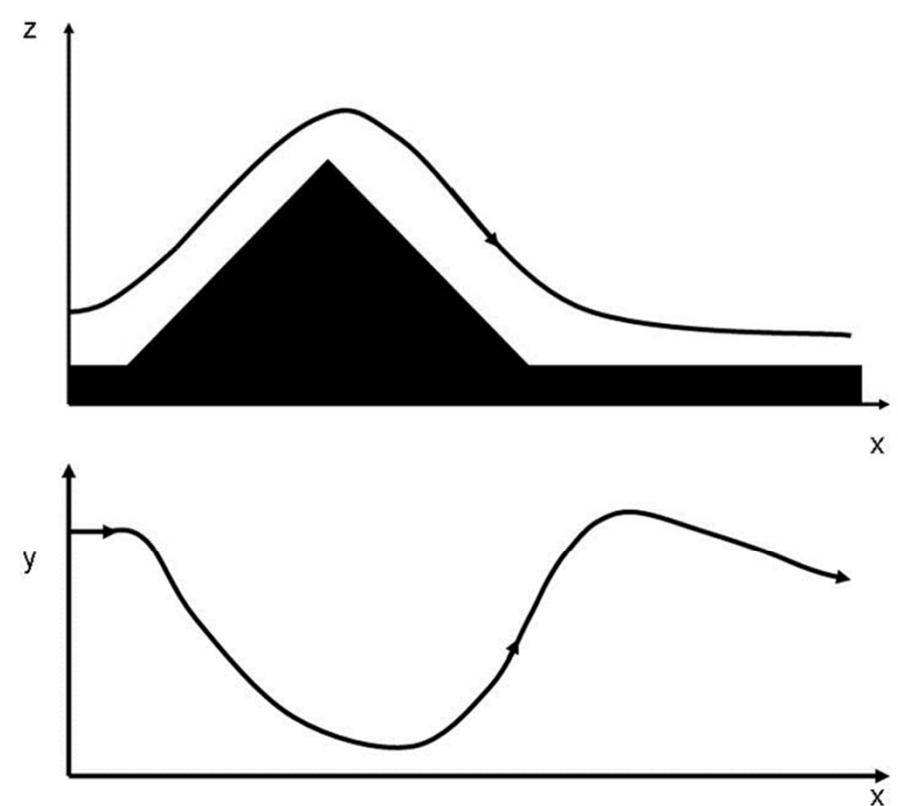
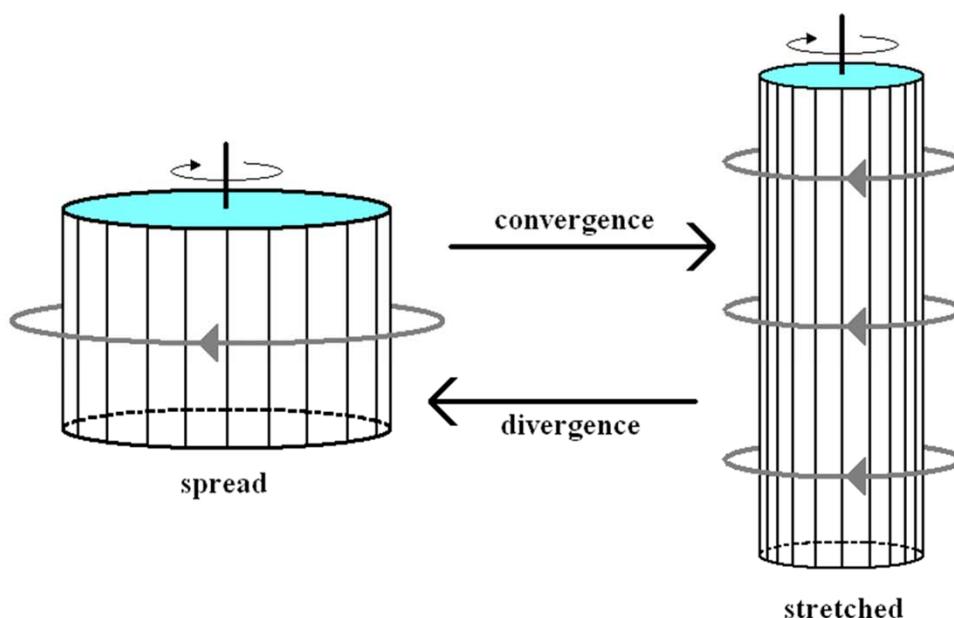
Path With Coriolis Effect

Conservations of potential vorticity

$$PV = \frac{1}{\rho} \zeta \cdot \nabla \theta$$

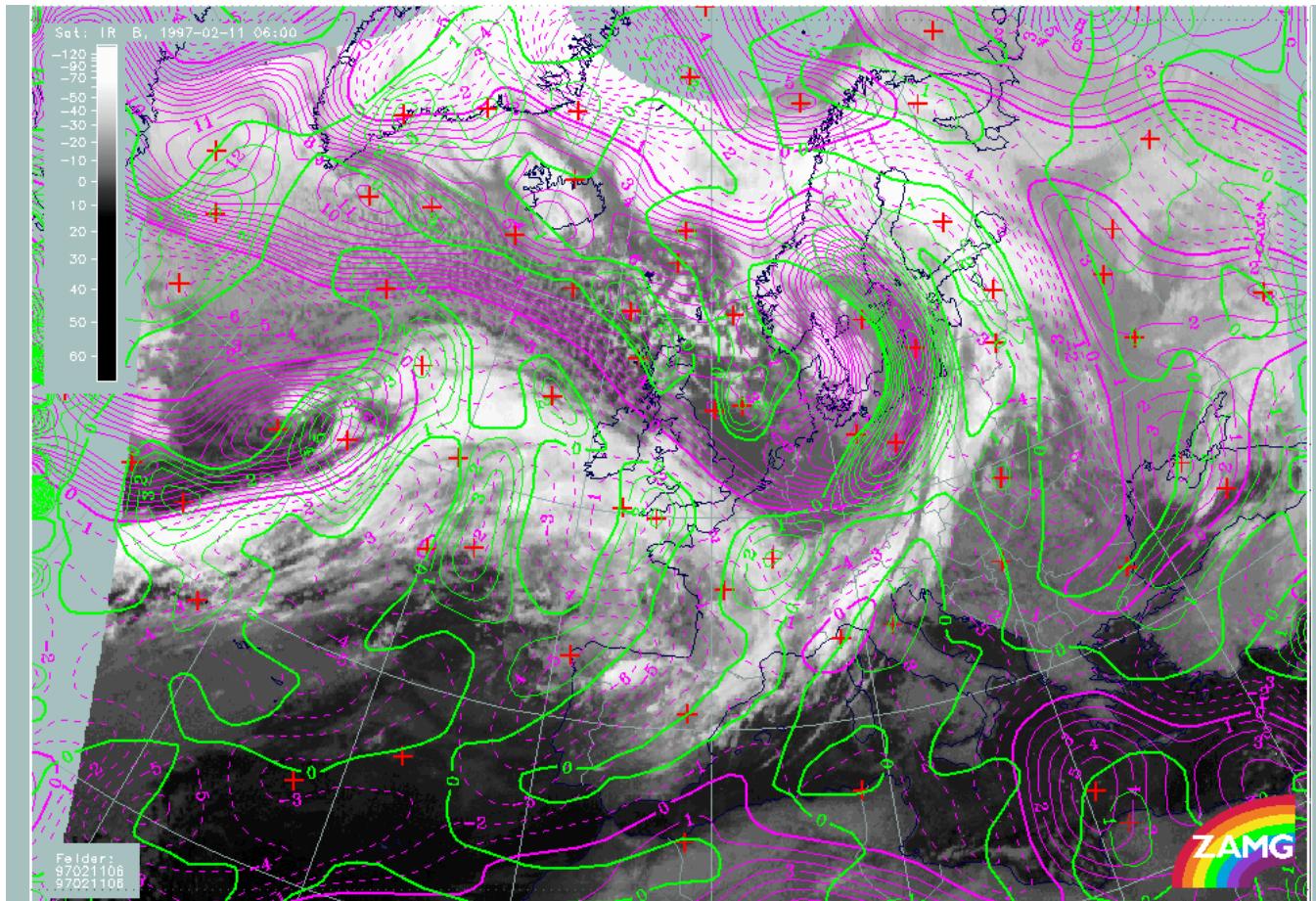
ρ , ζ , $\nabla \theta$ density, relative vorticity, potential temperature gradient

$$(\xi + f) \frac{\partial \theta}{\partial p} \equiv \text{constant}$$



Potential vorticity is useful in weather forecasting

Meteosat IR image of 11 February, 1997 0600 UT



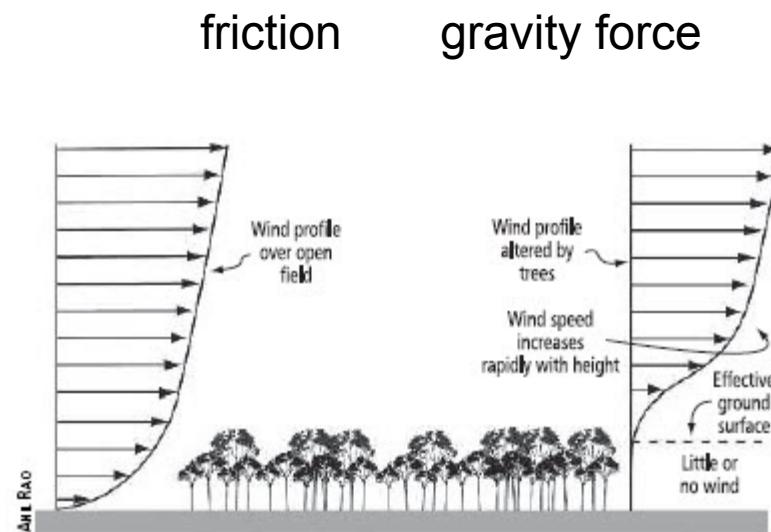
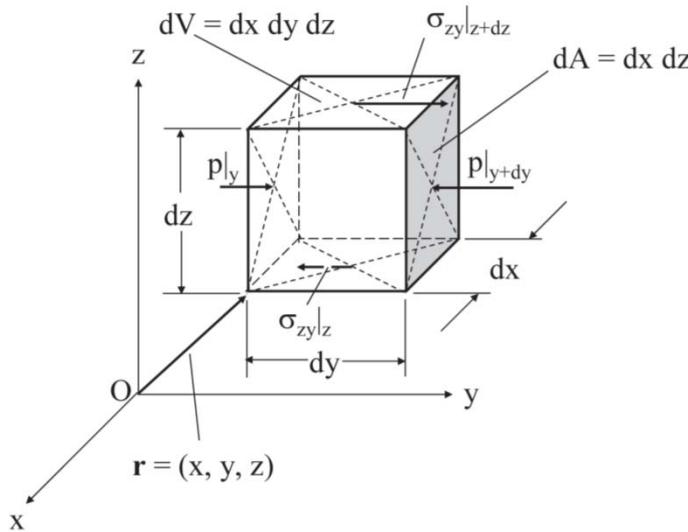
Magenta: relative vorticity at 500 hPa

Green: positive vorticity advection at 500 hPa

Navier Stokes Equation

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \frac{\eta}{\rho} \nabla^2 \mathbf{v} - \mathbf{g} - 2\Omega \times \mathbf{v}$$

Motion due to pressure gradient force friction gravity force Coriolis force



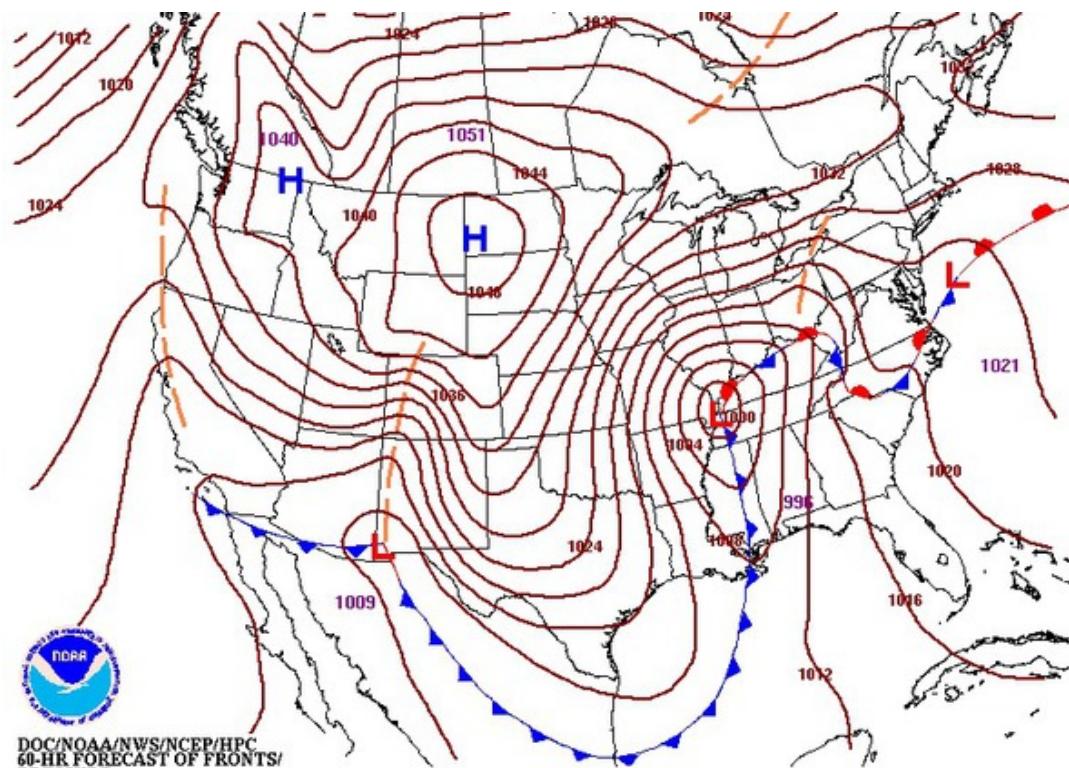
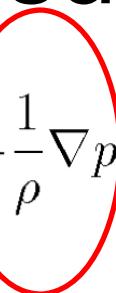
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Neglecting of the friction term leads to the Euler equation

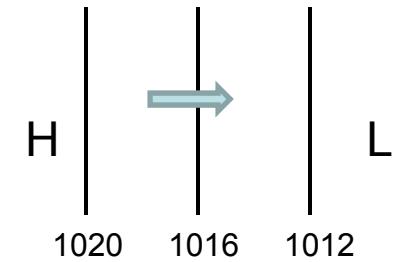
Most models use the Euler equation and parameterizations for friction in the ABL

Pressure gradient term

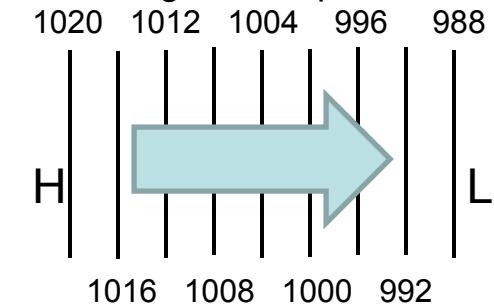
$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \frac{\eta}{\rho} \nabla^2 \mathbf{v} - \mathbf{g} - 2\Omega \times \mathbf{v}$$



Shallow pressure gradient:
low wind speed

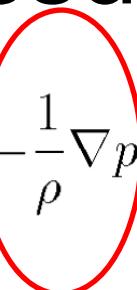


Step pressure gradient:
high wind speed

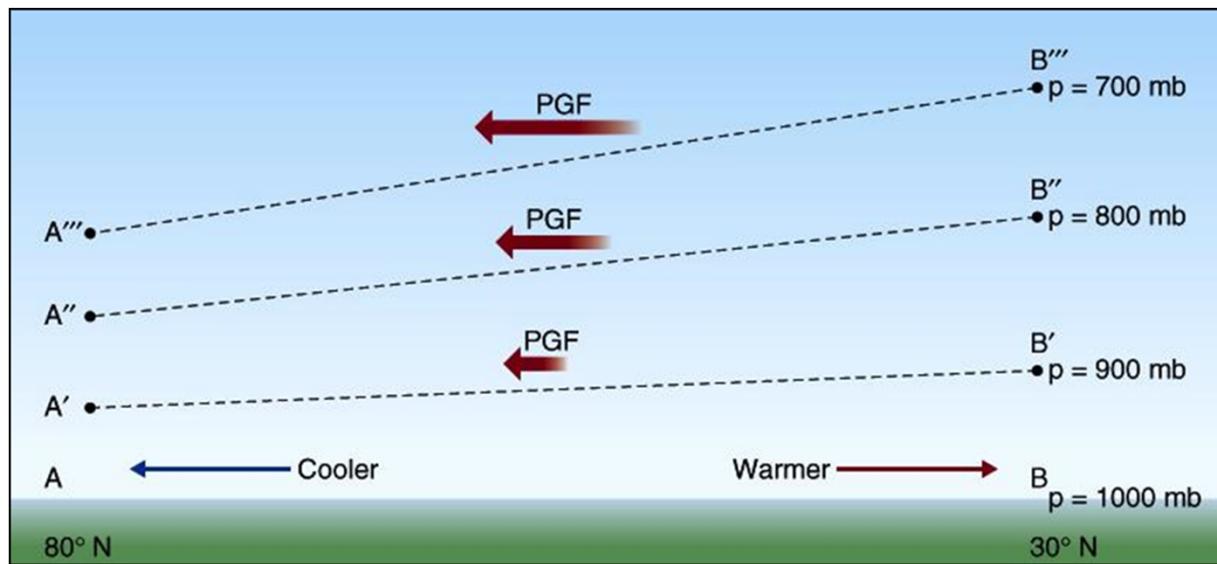
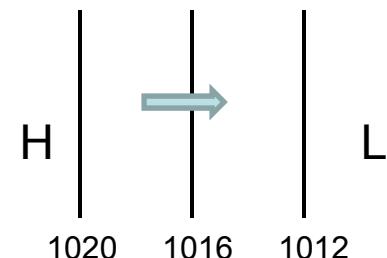


Pressure gradient term

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \frac{\eta}{\rho} \nabla^2 \mathbf{v} - \mathbf{g} - 2\boldsymbol{\Omega} \times \mathbf{v}$$



Shallow pressure gradient:
low wind speed



Step pressure gradient:
high wind speed

